

L10. Imperfect Competition: Workhorse Models

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EI037: Microeconomics, Fall 2021

Literature

- MWG, Chapter 12

Introduction

- In perfect competition models, all consumers and producers are assumed to act as price takers, in effect behaving as if the demand and supply functions that they face are infinitely elastic at going market prices.
- However, this assumption may not be a good one when there are only a few agents on one side of market, for these agents will possess **market power** – the ability to alter the profitable prices away from competitive levels.
- **Markets where agents' market power goes from high to low:** monopoly → oligopoly → monopolistic competition → perfect competition.
- In this lecture, we introduce the workhorse models characterising each of these markets. They (and their variations) constitute 99% of applied models you may confront in later studies.

Monopoly Pricing

When there is only a single seller in on market, we say it is a **monopolist** of that good.

- The monopolist's decision constitutes of choosing its price p so as to maximize its profits (in terms of the numeraire):

$$\text{Max}_p p \cdot x(p) - c(x(p))$$

an equivalent formation is choosing production quantity q , as $p = x^{-1}(q)$ anyway:

$$\text{Max}_q p(q) \cdot q - c(q)$$

- FOC (ignore corner solution):

$$p'(q^*)q^* + p(q^*) = c'(q^*)$$

⇒ the monopolist's optimal pricing (or quantity production) is such that its **marginal revenue of production equals marginal cost of production**

Static Models of Oligopoly

When there are more than one, but still not many, firms compete in one market, we say it is a market of **oligopolists**.

Competition among firms in an oligopolist market is inherently a setting of **strategic interaction**. The appropriate tool for its analysis is **game theory**.

- Recall a game has 4 basic elements. Depending on how we specify (or how the market looks like) for these elements we can have many variations.
- We only consider **1** environment: static games with perfect information; **2** models: Bertrand and Cournot.
 - In Bertrand competition firms strategically choose prices.
 - In Cournot competition firms strategically choose quantities.

The Bertrand Model of Price Competition

We begin with a case that there are two profit-maximization firms (a **duopoly**) in a market.

1. Their demand function is given by $x(p)$, with $x'(p) < 0$ and there exists a $\bar{p} < \infty$ such that $x(p) = 0$ for $p \geq \bar{p}$.
2. Their marginal cost of production is a constant c .
3. Two firms simultaneously name their price p_1 and p_2 . Sales for firm j are then given by

$$x_j(p_j, p_k) = \begin{cases} x(p_j) & \text{if } p_j < p_k \\ \frac{1}{2}x(p_j) & \text{if } p_j = p_k \\ 0 & \text{otherwise} \end{cases}$$

Proposition 10.1 There is a unique Nash equilibrium (p_1^*, p_2^*) in Bertrand duopoly model. In this equilibrium, both firms set their prices equal to the marginal cost: $(p_1^* = p_2^* = c)$.

Can you prove this? What is the intuition?

Show that with $J > 2$ number of firms, NE is all firms set their prices equal to c .

Remarks:

1. The striking implication of Proposition 10.1 is that with even only two firms we can get the perfect competitive outcome. Thus the Bertrand model predicts that the distortions arising from the exercise of market power are limited to the special case of monopoly.
2. Do you find this model capture (in some cases) the reality? Can you give some examples?
3. Can you think of any reason why this model may be unrealistic in some settings? Can you give some real world examples?

The striking result of Proposition 10.1 needs some strong (although very hidden) assumptions of the model setup.

1. Firms' strategic variable is price, not quantity (once we deviate from monopoly world, this matters)
2. There is no capacity constraint (one firm can serve the whole economy $x(p)$ as long as price she charges is lower than her rivals; there is no decreasing return to scale).
3. There is no production differentiation (consumers only care price, not who makes the product, as implied $x(p)$).

The Cournot Model of Quantity Competition

Two profit-maximization firms (a **duopoly**) in a market.

1. Demand function is given by $x(p)$ as before.
2. Marginal cost of production is a constant c as before.
3. Two firms simultaneously decide how much to produce, q_1 and q_2 . In equilibrium price adjusts to the levels that clears the market, $p = x^{-1}(q_1 + q_2)$.

To find a (pure strategy) NE, consider firm j 's maximization problem given an output level \bar{q}_k of the other firm

$$\text{Max}_{q_j} p(q_j + \bar{q}_k) \cdot q_j - c(q_j)$$

FOC:

$$p'(q_j^* + \bar{q}_k) \cdot q_j^* + q_j^* = c'(q_j^*)$$

Therefore in NE:

$$p'(q_j^* + q_k^*) \cdot q_j^* + q_j^* = c'(q_j^*); p'(q_k^* + q_j^*) \cdot q_k^* + q_k^* = c'(q_k^*)$$

When marginal cost is a constant c :

- In Cournot, market equilibrium satisfies

$$p'(q_1^* + q_2^*) \cdot \left(\frac{q_1^* + q_2^*}{2}\right) + p(q_1^* + q_2^*) = c$$

- Compared to in monopoly, market equilibrium is given by

$$p'(q^*)q^* + p(q^*) = c$$

Proposition 10.2 In any Nash equilibrium of the Cournot duopoly model with cost $c > 0$ per unit for the two firms and an inverse demand function $p(\cdot)$ satisfying $p'(q) < 0, p(0) > c$, the market price is greater than c (the competitive price) and smaller than the monopoly price.

Can you prove this? What is the intuition?

Remarks:

1. The presence of two firms is not sufficient to obtain a competitive outcome in the Cournot model, in contrast with the prediction of the Bertrand Model.
2. The key reason is that in this model, a firm no longer see itself facing an infinitely elastic demand.
3. → in Bertrand, a firm who increases price a bit at the margin can lose all demand; but in Cournot, a firm who increases quantity a bit will not decrease the price he face to zero.
4. → in Cournot, a firm is more like a monopoly with reduced market power.
 - when the number of firms goes to infinite, equilibrium is the same as that of perfect competition.
 - in the middle ground, we study an environment called monopolistic competition.

Monopolistic Competition

Monopolistic competition characterizes an industry in which many firms offer products or services that are similar, but are not perfect substitutes.

1. Products are imperfect substitutes hence all firms have market power.
2. But each firm is sufficiently small hence they take the market price as given.
→ Monopolistic competitive firms are “hybrides” between monopolists and perfect competitive firms

Examples?

We next study one popular model of monopolistic competition – [the Dixit–Stiglitz model](#).

The Dixit–Stiglitz Model

Product Differentiation

1. There are N (a large number) firms in the market, each produces one differentiated good.
2. A representative consumer has E income. His preference over N differentiated goods given by:

$$U = \left[\sum_{j=1}^N q_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} ; \sigma \geq 1$$

3. Given the utility function, the demand curve that is faced by firm j is given by

$$q_j = q_j(p_j) = \frac{p_j^{-\sigma}}{P^{1-\sigma}} E,$$

where P is the aggregate price index and $P^{1-\sigma} = \sum_{j=1}^N p_j^{1-\sigma}$.

Remarks:

i Note that the elasticity of substitution between two goods i and j is

$$\epsilon_{ij} = -\frac{d\ln(q_j/q_i)}{d\ln(p_j/p_i)} = \sigma.$$

- Hence the utility is called **constant elasticity of substitution (CES)** utility.

ii The elasticity of substitution across varieties is constant and given by σ :

- as $\sigma_i \rightarrow \infty$ and varieties become perfect substitutes.
- as $\sigma_i \rightarrow 1$ and we get the Cobb-Douglas case.

iii Notice the “love-of-variety” feature. Under perfect symmetry, *i.e.*, $q_i = q$ for all goods, and we have

$$U = N^{\frac{\sigma}{\sigma-1}} q = N^{\frac{1}{\sigma-1}} \times \underbrace{(Nq)}_{\text{“real” spending}} = N^{\frac{1}{\sigma-1}} E.$$

More varieties, greater the consumer utility.

4. Given the demand curve firm j face:

$$q_j = q_j(p_j) = \frac{p_j^{-\sigma}}{P^{1-\sigma}} E,$$

we know:

- firm j has some market power – the demand curve is not flat.
- but firm j is also sufficiently small – when she makes decisions, she takes aggregate price P as given instead of thinking her action can strategically influence P .

5. The profit maximization is therefore given by

$$\max_{p_j} q(p_j)p_j - q(p_j)c.$$

5. Solve firm j 's profit maximization problem, we get that firm charge constant markup over her marginal cost of production.

$$p_j^* = \frac{\sigma}{\sigma - 1} c.$$

Remarks:

- i Monopolistic competition model allows firm to have market powers, and at the same avoid to consider strategic interactions that can be very complicated.
- ii It captures some realistic features of incomplete market, at the same time maintains analytical solutions and share some elegance as perfect competition models.
- iii The Dixit–Stiglitz model has a wide application in trade and macroeconomics. Although it should be clear that the Dixit–Stiglitz model does *not equal to*, but belongs to the set of monopolistic competition models.
- iv Monopolistic competition models nevertheless also have their drawbacks. Can you think of any?

Perfect Competition

Although somewhat trivial, points below are helpful to think about the big picture regarding perfect competitive environment:

1. There are *infinite number of homogeneous* firms taking market price as given.
2. All firms charge $p = c$. (*note that constant return to scale assumption is necessary for perfect competition, why?*)
3. Note that the production or demand from “one particular firm” is undetermined under perfect competition, because firms are indifferent from consumer’s perspective.

To some extent, **there is no role of firms** in perfect competitive environment, firms’ problem is degenerated to **set price equal to marginal cost of production**, and **only aggregate variables matter** (i.e., aggregate demand $x(p)$ and market price p)!