

L7. The Fundamental Welfare Theorems

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Literature

- MWG (1995), Chapters 10D, 10E

This and the Previous Lecture

Previous Lecture: the organization of production and the allocation of the resulting commodities among consumers under *perfect competitive market economies*

→ positive perspective

Today's lecture: two central results regarding the **optimality** properties of competitive equilibria

→ normative perspective

Today's Lecture

The two central welfare results:

- If (i) there is a complete set of markets with publicly known prices, and (ii) every agent (firms, consumers) acts perfectly competitively (i.e., as a price taker):

The First Fundamental Welfare Theorem

- then the market outcome is Pareto optimal.

If additionally (iii) household preferences and firm production sets are convex:

The Second Fundamental Welfare Theorem

- then any Pareto optimal outcomes can be achieved as a competitive equilibrium if appropriate lump-sum transfers of wealth are arranged.

Notes on the Two Theorems

The First Fundamental Welfare Theorem

- when markets are complete (i), *any competitive equilibrium* (ii) is necessarily Pareto optimal
- Adam Smith's idea about the “invisible hand” of the market

The Second Fundamental Welfare Theorem

- w. + convexity conditions (iii)
- *all* Pareto optimal outcomes can be implemented through the market mechanism
- a public authority who wants one particular Pareto optimal outcome may do so by appropriately redistributing wealth and then “letting the market work”

Useful Benchmark

If any inefficiencies arise in a market economy, and hence any role for Pareto improving market intervention, we must be able to trace a violation of at least one of the assumptions of the **first fundamental welfare theorem**.

- Various market failures in reality: externalities, market power, info. frictions
- Can all be viewed as different themes developed (deviated) from the benchmark

Based on the **second fundamental welfare theorem**, if any inefficiencies arise in a market economy, one natural starting point of designing optimal policy is to “restore” perfect competitive environment and let “transfer + market” economy work.

- may not always be feasible, but a useful benchmark to start

- We prove both theorems under a partial equilibrium setting.
- We go through the general equilibrium intuition using the Edgeworth box.
- Check MWG (1995) chapter 15 if you are interested in the general treatment.

Proposition 7.1 (The First Fundamental Welfare Theorem) If the price p^* and allocation $(x_1^*, \dots, x_J^*, q_1^*, \dots, q_J^*)$ constitute a competitive equilibrium, then this allocation is Pareto optimal.

Proof under the partial equilibrium setting

Because of the quasilinear form of the utility functions, we have unlimited unit-for-unit transfer of utility across consumers through transfers of the numéraire (i.e. one unit of additional numéraire goods' utility gain is the same across consumers).

The set of utilities that can be attained for the I consumers is given by:

$$\{(u_1, \dots, u_I) : \sum_{i=1}^I u_i \leq \sum_{i=1}^I \phi_i(x_i) + w_m - \sum_{j=1}^J c_j(q_j)\}$$

The Pareto optimal allocation must involve quantities $(x_1^*, \dots, x_I^*, q_1^*, \dots, q_J^*)$ that max the RHS as much as possible \rightarrow If we “make the cake bigger”, there is always a way to achieve mutual benefits. Hence the necessarily condition is

$$\begin{aligned} \max_{(x_1, \dots, x_I) \geq 0, (q_1, \dots, q_J) \geq 0} & \sum_{i=1}^I \phi_i(x_i) + w_m - \sum_{j=1}^J c_j(q_j) \\ \text{s.t.} & \sum_{i=1}^I x_i - \sum_{j=1}^J q_j = 0 \end{aligned}$$

Lagrange equation:

$$\max_{(x_1, \dots, x_I) \geq 0, (q_1, \dots, q_J) \geq 0} \sum_{i=1}^I \phi_i(x_i) + w_m - \sum_{j=1}^J c_j(q_j) - u \left(\sum_{i=1}^I x_i - \sum_{j=1}^J q_j \right)$$

FOC:

$$D1 - \text{wrt. } q). \quad u \leq c'_j(q_j), \text{ with equality if } q_j^* = 0 \quad j = 1, \dots, J$$

$$D2 - \text{wrt. } x). \quad \phi'_i(x_i) \leq u, \text{ with equality if } x_i^* > 0 \quad i = 1, \dots, I$$

$$D3 - \text{wrt. } u). \quad \sum_{i=1}^I x_i^* = \sum_{j=1}^J q_j^*$$

D1-D3 are exactly parallel to conditions C1-C3, characterizing a competitive equilibrium (Lecture 6), with u replacing p^* !

Q.E.D.

Proposition 7.2 (The Second Fundamental Welfare Theorem of Welfare Economics)

For any Pareto optimal levels of utility (u_1^*, \dots, u_I^*) , there are transfers of the numéraire commodity (T_1, \dots, T_I) satisfying $\sum_i T_i = 0$, such that a competitive equilibrium reached for the endowments $(w_{m1} + T_1, \dots, w_{mI} + T_I)$ yields precisely the utilities (u_1^*, \dots, u_I^*) .

Proof under the partial equilibrium setting

Recall from the proof of the first fundamental welfare theorem, we showed that for any Pareto optimal levels of utility (u_1^*, \dots, u_l^*) , it must satisfy

$$D1. \quad p^* \leq c_j'(q_j), \text{ with equality if } q_j^* = 0 \quad j = 1, \dots, J$$

$$D2. \quad \phi_i'(x_i) \leq p^*, \text{ with equality if } x_i^* > 0 \quad i = 1, \dots, l$$

$$D3. \quad \sum_{i=1}^l x_i^* = \sum_{j=1}^J q_j^*$$

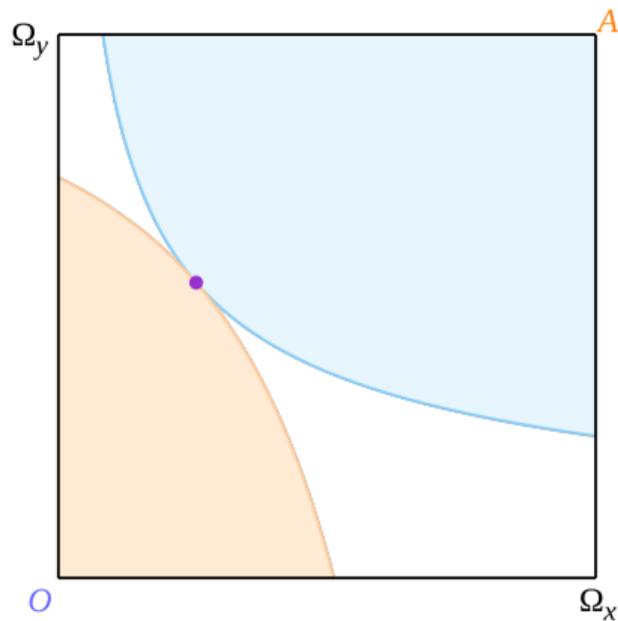
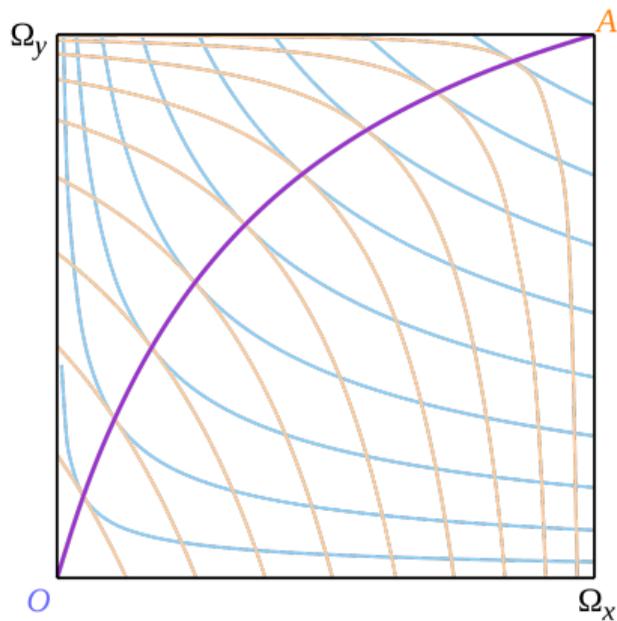
Hence under quasilinear preferences,

$$\begin{aligned} \sum_i u_i^* &= \sum_{i=1}^l \phi_i(x_i^*) + w_m - \sum_{j=1}^J c_j(q_j) \\ &= \sum_{i=1}^l (\phi_i(x_i^*) + w_{mi} + T_i - \sum_{j=1}^J \theta_{ij} c_j(q_j)) \end{aligned}$$

By construction the second line constitutes a Pareto optimal utility under transfer.

Edgeworth Box

First Fundamental Welfare Theorem



Edgeworth Box

Second Fundamental Welfare Theorem

