L1. Introduction, Preference and Choices

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EI037: Microeconomics, Fall 2021

Today's Lecture

- Course Details
- What is Microeconomics
- Preference and Choice

Course Details

Preambles

- Great to have feedback.
- Ask questions, slow me down when needed.
- Previews and reviews are important for studying micro-economic theory
- Expectations for econ vs. non-econ background students

Course Details

Organization of the Course

- Website
  - https://moodle.graduateinstitute.ch/course/view.php?id=2266
  - https://www.yuanzi-economics.com/ei037-microeconomics.html

- Main textbooks
  - "Microeconomic Theory", Andreu Mas-Collel, Michael D Whinston, and Jerry R Green, Oxford University Press (1995). [MMG]

- Additional textbooks
  - "Microeconomic Analysis", Hal R Varian, Norton (1992).
  - "A course in microeconomic theory", Kreps, David M, Princeton university press (1990).

- Grading
  - Midterm exam (1/3), and a final exam (1/3), as well as 2 problem sets ($2 \times 1/6$)
  - Problem set in groups (up to **5**), let TA know about members before the next review session

- Office hours:
  - Thursday 10-12 (course weeks)

Course Details

Outlines

- [Sep. 23] Preference and choice

  - How to define rational behavior?

- [Sep. 30, Oct. 7] Consumer theory, classical demand theory

  - What are the implications of rationality i.c.o. consumption theory?

- [Oct. 14] Aggregate demand and welfare economics

  - Which, if any, implications are preserved when aggregate individuals?

  - How to evaluate the welfare implications of economic policy changes?

- [Oct. 21] Production theory

  - What are the implications of rationality i.c.o. production theory?

$\rightarrow$ [October 28] Midterm exam

Course Details
Outlines

- [Nov. 4] Competitive equilibrium
  - How to understand market equilibrium?
- [Nov. 11] Welfare theorems
  - What are the implications of competitive equilibrium for welfare?
- [Nov. 18, 25] Strategic interactions
  - Deviate from 'atomic' agent
- [Dec. 2] Market power: workhorse models
  - Deviate from perfect competition?
- [Dec. 9] Uncertainty
  - How to describe decision problems under uncertainty?

$\rightarrow$ [Date TBA] Q&A, Final exam

Objectives

After the course, you should have developed a range of skills enabling you to understand economic concepts and use those concepts to analyze specific questions.

- Understand consumer behavior.
- Understand firm behavior.
- Analyze different types of market structures.
- Understand basic aspects of general equilibrium.
- Understand basic aspects of welfare economics.
- Understand how to apply economic principles to a range of policy questions.

What is Microeconomics?

Microeconomics (and a large part of modern macroeconomics as well) tries to explain **economic phenomena** as the outcome of **individual** decision making.

The focus of the analysis is always on the **individual**:

- What are the **objectives** and **constraints** of individual behavior?
- What should a **rational** individual do in a given situation?
- How does the **interaction** between individuals in markets and organizations shape economic outcomes?
- What are the **welfare** properties of an economic allocation (based on the welfare of all concerned individuals)?

Hence, the theory of individual decision making is of crucial importance for almost all subfields in economics.

Sometimes you may find the material of the course a bit frustrating: We will develop highly abstract and sophisticated theory in order to analyze individual decision making. Despite these efforts

- the immediate return in terms of testable implications is rather small;
- some empirical predictions that can be derived are falsified by experimental and field evidence.

However, you should keep in mind that this is due to the enormous **generality** of the theory that we develop. The more general a theory,

- the less structure it imposes on the model,
- the less testable empirical predictions it implies, and
- the easier it is to find counterexamples that contradict the theory.

Nevertheless, the generality of the theory of rational individual decision making makes it an extremely useful tool that can be applied to very different contexts.

If more restrictive assumptions (that stem from the specific context under consideration) are imposed, then we can get much more powerful and empirically testable results (Micro II, Trade etc..).

Preference and Choices

Literature

- MWG (1995), Chapter 1
- Kreps (1990), Chapters 1-2

- Experiment

Two Perspectives on Decision Theory

1. **Normative perspective:** What is a “rational” decision? If a decision maker has certain objectives and is facing certain constraints, which decision is optimal for him?
2. **Positive perspective:** How does actual decision making look like? Do actual decisions satisfy some “consistency” requirements?

These two perspectives correspond to two different approaches to modeling individual decision making

Preference-based approach

- takes the preferences of the individual as the starting point of the analysis
- defines “rationality” as assumptions on preferences
- solves the optimization problem of the decision maker in order to derive “decision functions” (e.g. demand functions).
- allows for welfare evaluations

Note: Preferences are not directly observable. They can only be elicited through introspection.

Choice-based approach

- takes the observed behavior of the decision maker as the primitive of the model
- makes assumptions directly on behavior (e.g., it requires that the behavior is “consistent” in some sense)
- does not allow for welfare evaluations.

Note: This approach does not speculate on unobservable preferences but is purely based on observed behavior. In principle it leaves room for more general forms of individual behavior.

At first glance these two approaches seem to be very different. However, we are going to show that they are very closely related to each other.

A Brief Description of the Two Approaches

Preference-Based Approach

The decision maker is supposed to have a **preference relation** $\succeq$ on the set of possible alternatives X.

$x \succeq y$ is read as: "x is at least as good as y" (for decision maker i)

- We can use the preference relation $\succeq$ to define

- the indifference relation $\sim$

$$x \sim y \Leftrightarrow x \succeq y \text{ and } y \succeq x$$

- the strict preference relation $\succ$

$$x \succ y \Leftrightarrow x \succeq y \text{ but not } y \succeq x$$

Preference-Based Approach

Definition 1.1 (Rationality)

The preference relation $\succeq$ is called **rational** if it possesses the following two properties:

(i) **Completeness:** for all $x, y \in X$, we have that $x \succeq y$ or $y \succeq x$ (or both)

(ii) **Transitivity:** for all $x, y, z \in X$, if $x \succeq y$ and $y \succeq z$, then $x \succeq z$.

- How strong are these assumptions?

- What do they imply for $\succ$ and $\sim$?

- It is difficult to work directly with preference relations. Therefore, it is very useful if $\succeq$ can be represented by a **utility function**.

Preference-Based Approach

Definition 1.2 (Utility function)

A function $u : X \to \mathbb{R}$ **represents** the preference relation $\succeq$ if, for all $x, y \in X$,

$$x \succeq y \Leftarrow u(x) \geq u(y)$$

Note:

- A utility function is a purely **ordinal** concept. It only gives a ranking of the different alternatives, but the numbers of the utility function have no **cardinal** properties.
- The utility function is **not unique**. For any strictly increasing function $f : \mathbb{R} \to \mathbb{R}$, $v(x) = f(u(x))$ is a new utility function that represents the same preference relation as $u(x)$. We say: “A utility function is unique up to a positive, monotone transformation.”
- **Interpersonal comparisons** of utility have no meaning.

The existence of a utility function is closely related to the assumption of rationality.

Proposition 1.1

A preference relation $\succeq$ can be represented by a utility function **only if** it is rational.

Note:

- What is the difference between "if" and "only if"?
- What do we have to prove?
- Can *any* rational preference relation $\succeq$ be described by some utility function?
    - In general the answer is no! (e.g., lexicographic preference defined over $\mathbb{R}^N$).
    - But when X is finite, the answer is yes.

The Proof:

s1. Note that direction of proof for "only if" is: "$\succeq$ can be represented by a utility function" $\Rightarrow$ "$\succeq$ is rational".

s2. According to the definition of "rational", it is equivalent to prove "$\succeq$ can be represented by a utility function" $\Rightarrow$ "$\succeq$" satisfies completeness and transitivity.

s3. Prove completeness:

$\because$ u(.) is a real-valued function defined on X.
$\therefore$ $\forall$ x, y $\in$ X, it must be that $u(x) \geq u(y)$ or $u(y) \geq u(x)$ (or both). [property of real-valued func.]
$\therefore$ $\forall$ x, y $\in$ X, we have that $x \succeq y$ or $y \succeq x$ (or both). [def. of that $\succeq$ can be represented by u(.)]
$\therefore$ $\succeq$ is complete. [def. of completeness]

s4. Prove transitive:
$\because$ u(.) is a real-valued function defined on X.
$\therefore$ $\forall$ $x, y, z \in X$, if $u(x) \geq u(y)$ and $u(y) \geq u(z)$, then:

1. $x \succeq y$ and $y \succeq z$. [def. of that $\succeq$ can be represented by u(.), e.g, $u(x) \geq u(y) \Rightarrow x \succeq y$]
2. $u(x) \geq u(z)$. [property of real-valued func.]

$\therefore$ $x \succeq z$ [def. of that $\succeq$ can be represented by u(.); $u(x) \geq u(z) \Rightarrow x \succeq z$]
i.e., $x \succeq y$ and $y \succeq z \Rightarrow x \succeq z$. $\succeq$ is transitive. [def. of transitive]

From s3 and s4, we proved $\succeq$ is complete and transitive. Hence $\succeq$ is rational. [def. of rationality]

Q.D.E.

Choice-Based Approach

A **choice structure** $(\mathcal{B}, C(\cdot))$ consists of two ingredients:

(i) $\mathcal{B}$ is a set of nonempty subsets of X, i.e., every element of $\mathcal{B}$ is a set $B \subset X$.

- B can be interpreted as the set of all sets from which the decision maker can choose (the set of all "budget sets").
- Note that $\mathcal{B}$ need not contain all subsets of X. Why don't we require this?

(ii) $C(\cdot)$ is a choice rule (a correspondence), that assigns a nonempty set of chosen elements $C(B) \subset B$ for every budget set $B \in \mathcal{B}$.

- Note that the set $C(B)$ may have more than one element.
- Note further that $C(B)$ is required to be non-empty. What does this mean?

Choice-Based Approach

Example: $X = \{x, y, z\}$ and $\mathcal{B} = \{\{x, y\}, \{x, y, z\}\}$.

- $C_1(\cdot) : C_1(\{x, y\}) = \{x\}, C_1(\{x, y, z\}) = \{x, z\}$
- $C_2(\cdot) : C_2(\{x, y\}) = \{z\}, C_2(\{x, y, z\}) = \{z\}$
- $C_3(\cdot) : C_3(\{x, y\}) = \{\phi\}, C_3(\{x, y, z\}) = \{x, z\}$
- $C_4(\cdot) : C_4(\{x, y\}) = \{x\}, C_4(\{x, y, z\}) = \{y\}$

Which of these choice rules are permitted? Interpret them.

How to think about “rationality” under the choice-based approach?

Definition 1.3 (weak axiom of revealed preference)

The choice structure $(\mathcal{B}, C(\cdot))$ satisfies the **weak axiom of revealed preference (WA)**, iff the following statement holds:

> If for some $B \in \mathcal{B}$ with $x, y \in B$ we have $x \in C(B)$, then for any $B' \in \mathcal{B}$ with $x, y \in B'$ and $y \in C(B')$, we must also have $x \in C(B')$.

- In words: If x is ever chosen when y is available, then there can be no budget set containing x and y in which y is chosen and x is not.

- Which one of the examples in the previous slide satisfies the weak axiom?

- The weak axiom is a consistency requirement that restricts choice behavior in a similar way as the rationality assumption in the preference-based approach.

To better understand the weak axiom, let us define the “revealed preference relation $\succsim^*$ from the observed choice behavior in $C(\cdot)$:

Definition 1.4 (revealed at least as good as $(\succsim^*)$)

x is **revealed at least as good as** y ($x \succsim^* y$) if and only if there is some $B \in \mathcal{B}$ such that $x, y \in B$ and $x \in C(B)$.

- Note that $\succsim^*$ need not be complete nor transitive.

- Note “if and only if”.

Using this definition we may say that:

"x is **revealed preferred to** y" if for some $B \in \mathcal{B}$ with $x, y \in B$ we have that $x \succeq^* y$ and **not** $y \succeq^* x$.

With this terminology, we can restate the weak axiom as follows:

> **Weak Axiom of Revealed Preference:** If x is revealed at least as good as y, then y cannot be revealed preferred to x.

Relationship between Preference Relations and Choice Rules

- Suppose that a decision maker has a rational preference ordering $\succeq$. Does this imply that her decisions when facing choices from budget sets in $\mathcal{B}$ necessarily generate a choice structure that satisfies the weak axiom?
- In order to answer this question we have to derive the choice rule $C(B)$ from the preference relation $\succeq$:
- Let $C^*(B, \succeq) = \{x \in B | x \succeq y \text{ for every } y \in B\}$, i.e., if the decision maker faces a non-empty set of alternatives $B \subset X$, then her preference maximizing behavior is to choose all elements of B that are not strictly preferred by some other element of B.
- The following proposition shows that rationality of the preference ordering implies the weak axiom.

Proposition 1.2 (rational preference $\Rightarrow$ WA)

Suppose that $\succeq$ is a rational preference relation. Then the choice structure generated by $\succeq$, $(\mathcal{B}, C^*(\cdot, \succeq))$ satisfies the weak axiom.

Relationship between Preference Relations and Choice Rules

Suppose now that an individual's choice behavior satisfies the weak axiom. Does this imply that she has a rational preference relation $\succeq$ that "rationalizes" her choices?

PS. We say that a rational preference relation $\succeq$ **rationalizes** $C(\cdot)$ relative to $\mathcal{B}$ iff $C(B) = C^*(B, \succeq)$ for all $B \in \mathcal{B}$.

$\rightarrow$ In general the answer to this question is **No!**

Relationship between Preference Relations and Choice Rules

To see this, consider the following **example:**

- $X = \{x, y, z\}$
- $\mathcal{B} = \{\{x, y\}, \{y, z\}, \{x, z\}\}$
- $C(\{x, y\}) = \{x\}, C(\{y, z\}) = \{y\}, C(\{x, z\}) = \{z\}$

Verify that this choice structure satisfies the weak axiom. Nevertheless, we cannot have rationalizing preferences. Why?

The problem with this example is that the set $\{x, y, z\} \notin \mathcal{B}$. (Why don't we require this?)

However, the following proposition shows that if $\mathcal{B}$ includes "enough" subsets of X, then the answer to our second question is **Yes!**

Relationship between Preference Relations and Choice Rules

Proposition 1.3 (Arrow, 1959)

If $(\mathcal{B}, C(\cdot))$ is a choice structure such that

- the weak axiom is satisfied, and
- $\mathcal{B}$ includes all subsets of X of up to three elements,

then there exists *exactly one* rational preference relation $\succeq$ that rationalizes $C(\cdot)$ relative to $\mathcal{B}$.

The Proof (MWG, Proposition 1.D.2.):

A. Prove Existence:

A.1. Prove $\succeq^*$ is complete.

$\because$ $\mathcal{B}$ includes all subsets of X of up to three elements

$\therefore$ $\forall$ x, y $\in$ X, there exist one budget set $B \in \mathcal{B}$ such that only $x, y \in$ B.

$\because$ $C(.)$ can't be empty, hence we must have that $x \succeq^* y$ or $y \succeq^* x$ (or both).

$\therefore$ $\succeq^*$ is complete.

A.2. Prove $\succeq^*$ is transitive.

$\because$ $\mathcal{B}$ includes all subsets of X of up to three elements

$\therefore$ $\forall$ x, y, z $\in$ X, there exist at least one budget set $B' \in \mathcal{B}$ such that x, y, z $\in$ B'.

Note that the proved A.1 ensures that all pair-wise comparison exits. Without loss of generality suppose $x \succeq^* y$, $y \succeq^* z$. Then weak axiom requires that $x \in C(B')$. This in turn implies that $x \succeq^* z$ according to the definition of revealed preference.

$\therefore$ $x \succeq^* y$, $y \succeq^* z \Rightarrow x \succeq^* z$, i.e., $\succeq^*$ is transitive.

Let $\succeq$ being $\succeq^*$. Then $\succeq$ rationalize C(.) relative to $\mathcal{B}$. Q.D.E.

B. Prove Uniqueness:

Simply note that because $\mathcal{B}$ includes all two-elements subsets of X, the choice behavior in C(.) completely determines the pairwise preference over X for any rationalizing preference.

(note 1: preference is ordinary concept; note 2: this proposition says any choice rule (B, C(.)) satisfy condition 1.2. has one (and only one) rational relation corresponding to it; it does not say there is only one choice rule that can be rationalized by preference approach, don't get confused!)

Conclusions

1. If we can observe the individual's choice behaviour for **all possible subsets** of X, then the rationality assumption of the preference-based approach and the weak axiom of the choice-based approach are completely equivalent.

2. However, in reality we only observe an individual's choices for **some subsets** of X. In this case the rationality assumption implies the weak axiom but not vice versa.