

International Trade – Fall 2019

Problem Set 4

Melitz Model

The model is the same as the one discussed in class, all the notations are the same as in the lecture's slides. Wages are fixed and normalized to 1.¹

1. First consider a closed economy.

- a. The equilibrium is defined by three variables: the economy's average profit $\bar{\pi}$, the cutoff productivity $\bar{\varphi}$, and the mass of firms M (i.e. the number of firms). Explain how the equilibrium is determined. Show that $M = \frac{L}{\sigma(\bar{\pi}+f)}$. Comment briefly.
- b. Show that welfare per worker can be expressed as $W = \frac{\sigma-1}{\sigma} M^{\frac{1}{\sigma-1}} \tilde{\varphi}$. Then show that we can express W as a function of the productivity cutoff $\bar{\varphi}$ as: $W = \frac{\sigma-1}{\sigma} \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \bar{\varphi}$.

Plug $r(\tilde{\varphi}) = \sigma f \left(\frac{\tilde{\varphi}}{\bar{\varphi}}\right)^{\sigma-1}$ into (??), after some algebra one can show that $W = \frac{\sigma-1}{\sigma} \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \bar{\varphi}$.

2. Now consider an open economy. There are two symmetrical countries trading with each other. Firms in each country face an iceberg trade cost $\tau > 1$ and a fixed cost f^X to export to the other country. As in the lecture's slides, $\bar{\varphi}$ is the productivity cutoff such that domestic profits are zero, and $\bar{\varphi}^X$ the productivity cutoff such that export related profits are zero. Consider the equilibrium in the home country.

- a. Show that $\bar{\varphi}^X = \tau \left(\frac{f^X}{f}\right)^{\frac{1}{\sigma-1}} \bar{\varphi}$. Why is it realistic to assume that $\tau \left(\frac{f^X}{f}\right)^{\frac{1}{\sigma-1}} > 1$?
- b. Show that $M = \frac{L}{\sigma(\bar{\pi}+f+prob^X f^X)}$. Compare with question 1.(a), and comment.
- c. Express the total number of varieties available to consumers in the home country under trade (denote by M_T), as a function of M .

¹Note that instead of assuming this, we could assume, as in Krugman and Helpman (1985), that there is a second sector producing a freely traded homogenous good under perfect competition and constant returns to scale, and that productivity is normalized to 1 in this sector in both countries. Wages will be pinned down by productivity and equalized across countries.

- d. By analogy with question 1.(b), one can show that $W = \frac{\sigma-1}{\sigma} M_T^{\frac{1}{\sigma-1}} \tilde{\varphi}_T$ where $\tilde{\varphi}_T$ is the average productivity of *all* the firms selling in the home country. Express W as a function of $\bar{\varphi}$. Compare with expression found in 1.(b) in the closed economy case and comment briefly.