Trade Costs, Global Value Chains and Economic Development*

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May 4, 2017

Abstract

This paper develops a model to study the impact of trade costs on developing countries' industrialization when sequential production is networked in global value chains (GVCs). In a two-country setting, a decrease in trade costs of intermediates is associated with South joining and moving up the value chain and both North and South experiencing welfare improvement. The wage gap between North and South first increases and then decreases. Extending the model to a multi-country setting, I show that reduced trade frictions lead South countries to join GVCs due to wage differentials and low trade costs. This increases the wage in North but may decrease the wages of South nations that are already part of the network. Moreover, South nations that join tend to be regionally clustered. The model provides a first look at GVCs from the development angle, and raises policy questions regarding the governance of GVCs.

JEL Classification: F10, F11, F15
Keywords: trade costs, industrialization, joining and moving up the value chain

*I am grateful to Richard Baldwin, Nicolas Berman, Arnaud Costinot, Davin Chor, Aksel Erbahar, Bart Los, Ugo Panizza, Frédéric Robert-Nicoud, Stela Rubínová, Su Wang and Yoto Yotov for helpful comments at various stages of the development of this paper. Additionally, I thank the faculty of the Massachusetts Institute of Technology (MIT) for their hospitality during my visit as a doctoral researcher.

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1 Introduction

Driven by the information and communication technology (ICT) revolution and other forms of technological progress, firms and countries nowadays increasingly specialize in stages of value chains rather than producing final goods from scratch. This increased international production fragmentation is changing the landscape of international trade, reshaping scholars' and policymakers' understanding of development policy, optimal trade policy, and the impact of trade on firms and labor. One observation that has received much attention is the revolutionized development options faced by poor nations – now they can join value chains rather than having to invest in building their own, a process which can take decades (Baldwin, 2011). But how can a developing country join value chains, and how might that affect its wage, production complexity, and income gap with developed countries? If global trade costs fall steadily due to the ICT revolution, which developing country will join GVCs first? Who follows? How will the production and welfare of countries that are already part of value chains be affected by the new joiners? These are intriguing policy questions that need theoretical underpinnings.

This paper tries to fill this gap by developing a simple model to extend the work by Costinot et al. (2013) by studying the impact of iceberg trade costs on developing countries' industrialization. I begin with a two-country setting, North and South, in Section 2. There are two alternative ways of organizing production: modern and traditional. Modern production involves sequential production à la Costinot et al. (2013), with firms specializing in one stage of the production process. Traditional production requires no coordination between firms: each firm produces the final good using labor as the only input. Suppose initially that only North has modern production and there is no production fragmentation across borders. The model predicts that the "industrialization" of South consists of two episodes as trade costs in intermediates fall. In the first episode, labor in South shifts from traditional to modern production à la Lewis (1954). However, there is no real wage improvement in South, even though it produces in increasingly complicated stages of modern production. Wage inequality between North and South rises. The second episode starts when modern production in South employs the entire labor force. In this episode, a decrease in trade costs is associated with an increase in real wages in both countries and a decrease in global inequality. In both episodes, South moves up the value chain, and North experiences an industrial hollowing-out as trade costs fall.

Section 3 extends the model to a multi-country setting. Conditional on participation, countries are strictly ordered along stages by their productivity in each value chain. In a one-North-multi-South setting, when global trade costs decline, South countries form value chains with North sequentially. If all South countries have the same modern technology, the production network exhibits a hub-and-spoke shape. The joining of a new South decreases the welfare of other South countries inside the value chain, although every South country moves up the value chain. If learning-by-doing exists, the model generates a flying geese pattern of development: South countries join the value chain sequentially with the most recent joiner specializing at the most upstream stages of the value chain. If the learning effect is strong enough, the joining of a new South benefits every country inside the value chain. In

both cases, the successful joining of new countries depends on their proximity to South countries that are already part of the value chain, implying factory economies are likely to be regionally clustered.

This paper is motivated by recent discussions on how GVCs has transformed the development strategy and trade policy of developing countries. Important works include Gereffi et al. (2005), Baldwin (2011) and Baldwin (2012). My paper contributes to this strand of the literature by providing a tractable theoretical framework that formalizes the analysis. This paper is also inspired by recent empirical literature on supply-chain trade, such as Hummels et al. (1998), Hummels et al. (2001), Koopman et al. (2014) and Johnson and Noguera (2012b; 2012a), and has been able to confirm several empirical regularities that have been documented in the literature.

In terms of focus, this paper is related to the literature on sequential production in an international context. Important works include Findlay (1978), Sanyal and Jones (1982), Dixit and Grossman (1982), Spencer and Jones (1992) and Costinot et al. (2013). These studies examine the impact of shock transmissions, technological changes and trade policies on countries' welfare, specialization patterns, trade flows and global inequalities. Among others, Yi (2003), Harms et al. (2012) and Baldwin and Venables (2013) analyze the non-monotonic trade response to reductions in trade costs, Kohler (2004) stresses the impact of outsourcing on factor prices in the outsourcing economy. Important papers such as Antràs and Chor (2012) and Kikuchi et al. (2012), Fally and Hillberry (2015) discuss firm boundaries and the slicing of value chains from a property-rights and Coasian perspective, respectively. My paper is closely related to Yi (2003), Harms et al. (2012) and Baldwin and Venables (2013) as I also focus on the implications of trade costs on vertical specialization and trade flows.

In terms of modeling techniques, this paper is closely related to the hierarchies literature with studies by, for instance, Sobel (1992), Kremer (1993) and Garicano (2000). It is also related to the literature on assortative matching in an international setting; see Yeaple (2005), Ohnsorge and Trefler (2007), Nocke and Yeaple (2008), Costinot (2009) and Costinot et al. (2013), among others. In particular, my model is mostly built on Costinot et al. (2013). My analysis is, however, novel across two dimensions. First, I introduce standard iceberg trade costs in intermediates, so countries can be excluded from GVC participation due to their geographical disadvantages. Second, I assume that developing countries have no modern production to start with, which puts GVC participation and economic development at the center of the analysis. As a result, I am able to add to previous contributions by formally analyzing how "the second global unbundling" (Baldwin, 2006) benefits developing nations by enabling them to industrialize through joining GVCs.

The rest of the paper is organized as follows. In the next section, I present the two-country model and analyze the impact of trade costs on wages, production specialization and inequality between North and South. Section 3 presents the multi-country extensions. Section 4 discusses policy implications of the paper, and Section 5 concludes. Mathematical proofs are relegated to the Appendix.

2 A Model of Sequential Production with Trade Frictions

I extend Costinot et al. (2013) to incorporate iceberg trade costs in intermediates.[1] To emphasize the new insights generated from the model, I restrict my attention to a two-country setting in this section.

2.1 Basic Setup

I consider a world with two countries, North (N) and South (S), one factor of production, labor, and one final good. Labor is inelastically supplied and immobile across countries. L_c and w_c denote labor endowment and wage in country $c \in \{N, S\}$, respectively.

There are two ways of organizing production: traditional and modern. Traditional production involves a single firm producing one unit of the final good using a_c units of labor. Modern production requires firms to specialize and to produce at an infinitesimal stage of a production chain. As in Costinot et al. (2013), to produce the final good, a continuum of stages $s \in (0, 1]$ must be performed sequentially. Production of one unit of intermediate good at a stage requires one unit of labor and one unit of the intermediate good produced in the previous stage. Firms are prone to mistakes when combining intermediates with labor to produce goods for the next stage. Mistakes occur at an exogenous, country-specific Poisson rate λ_c. Formally, without crossing borders, the output of intermediate good at stage $s + ds$ (with ds infinitesimal) is given by:

$$q_c(s + ds) = (1 - \lambda_c ds) q_c(s). \tag{1}$$

Without loss of generality, I assume that North is more productive in modern production, $\lambda_S > \lambda_N$; both countries are equally productive in traditional production, $a_S = a_N = a$, with $\frac{1}{a} < \frac{\lambda_N}{e^{\lambda_N} - 1}$. I refer to λ as the measure of (modern) productivity in the rest of my analysis.

All markets are perfectly competitive. The price of intermediate good at stage s in country c is denoted as $p_c(s)$. For expositional purposes, I assume that the intermediate good at stage 0 has infinite supply and zero price, $p_c(0) = 0$. The final good, or "intermediate good at stage 1", is freely traded and its price is set as the numeraire, $p_c(1) = 1$. For technical reasons as described in Costinot et al. (2013), I further assume the existence of $s_{\Delta s} < s < s_{\Delta s} + \Delta$ such that if $q(s) > 0$, then $q(s') > 0$ for $s' \in (s_{\Delta s}, s_{\Delta s} + \Delta]$.

Admittedly stylized, my assumption on traditional versus modern production captures the idea that, in reality, many goods can be produced using different technologies. In some less developed regions in Africa and Asia, for example, textile is largely a cottage industry, with family members working in their home-transferred workspace; while in more developed countries, yarning, dying, finishing, and fabric milling are ran by different and specialized firms (Cetindamar et al., 2005).

[1] Costinot et al. (2013) studied the implications of "coordination costs," which are modeled as defect rates in final goods that increase in the *number* of times the intermediates used in production have crossed a border. As it will become clear in the next section, unlike coordination costs, which are determined only by the number of times the intermediates have crossed a border, and directly affect only the output of final goods, the presence of iceberg trade costs gives distinct predictions regarding countries' specialization and welfare.

Note that this does not rule out the possibility that the traditional production is sequential. It only requires that traditional goods are more economical when produced within a firm's boundary, and that semi-finished goods from transitional production cannot be used for modern production.

Trade in intermediates is subject to iceberg costs. Formally, when stage s and $s + ds$ are located in different countries, say countries c and c' respectively, equation (1) becomes:

$$q_{c'}(s + ds) = \frac{1}{\tau}(1 - \lambda_{c'} ds) q_c(s), \tag{2}$$

where $\tau \in [1, \infty)$, with $\tau = 1$ indicating frictionless production-sharing across countries. Broadly speaking, τ captures all costs associated with coordinating value chains. In addition to the standard trade costs, it also reflects productivity losses because of costly communication, language and cultural differences, back-and-forth travel by key personnel, etc.

2.2 Equilibrium Productions

In equilibrium, all firms maximize profits taking the world price as given and all markets clear. As the price of the final good is the numeraire, if traditional production exists in country c, wage w_c will be pinned down at $\frac{1}{a}$. As for modern production, since all markets are perfectly competitive, profit maximization requires that for all $c \in \{N, S\}$,

$$\begin{aligned} p_c(s + ds) \leq [(1 + (\tau - 1)\mathbb{1}_{c \neq c'})(1 + \lambda_c ds) p_{c'}(s) + w_c ds] \text{ for both } c' \in \{N, S\}, \\ \text{with equality holds for at least one } c' \text{ if } Q_c(s') > 0 \text{ for all } s' \in (s, s + ds], \end{aligned} \tag{3}$$

where $Q_c(s')$ denotes total output of modern production at stage s' in country c and $\mathbb{1}$ is an indicator function that equals one when $c \neq c'$. Condition (3) states that, taking into account trade costs, the price of intermediate good $s + ds$ must be weakly less than their unit costs of production, with equality if it is actually produced in country c.

Goods and labor market clearing further require that:

$$\sum_{c \in N,S} Q_c(s2) - \sum_{c \in N,S} Q_c(s1) = -\int_{s1}^{s2} \sum_{c \in N,S} \Omega_{cs}(\mathbb{P}) \lambda_c Q_c(s) ds, \tag{4}$$

$$\int_0^1 Q_c(s) ds + a Q_c^T = L_c, \text{ with } w_c \geq \frac{1}{a} \text{ and } Q_c^T = 0 \text{ if } w_c > \frac{1}{a}, \tag{5}$$

where Q_c^T denotes the total output of traditional production in country c, and $\Omega_{cs}(\mathbb{P})$ is a trade "haircut" depending on the trade costs and international production-sharing structure $\mathbb{P}$. For instance, $\Omega_{cs} = 1$ if $Q_c(s)$ are all used by domestic downstream users, and $\Omega_{cs} = \frac{\tau + \lambda - 1}{\tau \lambda}$ if $Q_c(s)$ are all used by foreign downstream users. An equilibrium therefore corresponds to output levels Q^T, $Q_c(s)$, intermediate goods prices $p_c(s)$, wage w_c for $c \in \{N, S\}$ and $s \in (0, 1]$, and an international production-sharing structure $\mathbb{P}$ such that conditions (3) – (5) hold.

Next, I discuss all possible equilibrium production structures. When trade costs are infinitely high, there will be no production-sharing between North and South. I refer to such equilibrium as "separate-production" equilibrium. In this situation, depending on the value of a and λ_c, the two countries can have either modern or traditional production. As each stage is infinitesimal and there is no strategic interaction among firms, a country only engaging in traditional production is clearly a stable equilibrium. In this case, the country's equilibrium wage and final good production are:

$$w_c = 1/a, \; Q_c = L_c/a. \tag{6}$$

When does modern industry exist in a separate-production equilibrium? Notice that if one country has all stages of modern production, equation (3) requires that, for any stage s, the following price equation must hold:

$$p_c(s + ds) = (1 + \lambda_c ds)p_c(s) + w_c ds. \tag{7}$$

Equation (7), together with the boundary conditions $p_c(0) = 0$ and $p_c(1) = 1$, implies that the equilibrium wage in country c must be equal to:

$$w_c = \frac{\lambda_c}{e^{\lambda_c} - 1}. \tag{8}$$

Notice that for modern production to exist, the wage offered must be higher than the wage offered under traditional production, i.e., $\frac{1}{a} < \frac{\lambda_c}{e^{\lambda_c}-1}$. Otherwise, the traditionally produced final good will be cheaper; hence, firms always have the incentive to deviate.

Next, I characterize production-sharing equilibrium. In general, because of the interdependence of firms in modern production, the range of production stages that a country operates in depends on its initial industry structure. When each stage is infinitesimal, I am able to circumvent the industry's lumpiness and obtain relatively simple results. If there is any production-sharing between North and South, it must be that South supplies upstream intermediates to North. This result is summarized in the following proposition:

PROPOSITION 1. If international production-sharing exists, it must be that there exists a cutoff stage $\tilde{s} \in (0, 1)$ such that $Q_S(s) > 0$ if $s \in (0, \tilde{s}]$ and $Q_N(s) > 0$ if and only if $s \in (\tilde{s}, 1]$. Traditional production may coexist with modern production in South.

The intuition behind proposition 1 is along the lines of Sobel (1992), Kremer (1993), and Costinot et al. (2013). As price increases along the value chain and the defect rate is always proportional to the price of intermediates, it is relatively more costly to make mistakes in the later stages. Hence North, with a lower defect rate, has a comparative advantage in producing downstream goods.

When production-sharing exists, as shown in the Appendix, the wage between North and South must satisfy the following condition:

$$w_N = \tau w_S + \tau p(\tilde{s})(\lambda_S - \lambda_N). \tag{9}$$

I define the wage gap between North and South as $\frac{w_N - w_S}{w_S}$. From equation (9), we see that the wage gap is an increasing function of trade cost τ and the technology gap between the two countries. Trade costs act as a wedge between the wage in South and North: the higher the trade cost, the lower the price South must charge for its intermediates to attract North downstream buyers, which in turn implies a lower wage in South. All else equal, an increase in $p(\tilde{s})$ also implies a rise in the wage gap. The intuition is simple: higher $p(\tilde{s})$ means that intermediates are more costly when shipped from South to North. Therefore, South wage has to be low enough to make production-sharing *at this stage* attractive to North.

2.3 Reduction in Trade Costs

Since the 1980s, trade costs have decreased significantly thanks to rapid technological progress. Cheaper and more reliable telecommunication facilities have made coordinating complex activities across borders easier and more timely; advancement in computer software made the development of international multimodal transport systems and "door-to-door" transportation possible, which greatly facilitated the fragmentation of production across country borders. In this section, I use the model to analyze how the reduction in trade costs transforms the development opportunities for South. I focus on the following thought experiment: suppose initially only North has modern production and there is no production fragmentation between North and South due to high trade costs in intermediates. When trade costs decrease, how do countries' real wages and specialization patterns respond?

I start from the separate-production equilibrium where South has only traditional production and North has modern. In this equilibrium, wages in the two countries are $w_S = \frac{1}{a}$ and $w_N = \frac{\lambda_N}{e^{\lambda_N}-1}$, respectively. Since $\frac{1}{a} < \frac{\lambda_N}{e^{\lambda_N}-1}$, there is a wage difference between North and South. When trade costs become low enough, South will find it profitable to produce intermediates to serve downstream firms in North. Note that this change cannot start from an "interior" stage, at which South firms both source from and produce for North. Since each stage is infinitely small, the cost advantage of firms in South are outweighed by the trade costs. In other words, production unbundling can start only from the most upstream stage of the value chain. Given this, it is easy to find the cutoff trade cost $\tau_1 = \frac{a\lambda_N}{e^{\lambda_N}-1}$, at which North firms using the most upstream intermediates are indifferent between sourcing domestically or from South.

When production-sharing happens in this case, from proposition 1, we know that it must be South that specializes in the upstream of the value chain and North in the downstream. Given equations (7) and (9), intermediates' price at each stage can be expressed as a function of wages w_N, w_S, and cutoff stage $\tilde{s}$:

$$p(s) = (e^{\lambda_S s} - 1)\frac{w_S}{\lambda_S},\ for\ s \leq \tilde{s}, \tag{10}$$

$$p(s) = e^{\lambda_N(s-\tilde{s})}\tau p(\tilde{s}) + (e^{\lambda_N(s-\tilde{s})} - 1)\frac{w_N}{\lambda_N},\ for\ \tilde{s} < s \leq 1. \tag{11}$$

The price of intermediates in South is negatively related to South productivity (inverse of λ_s) and positively related to wage and the complexity of the intermediate stages. Intermediates in later stages

require more value-added along the value chain and are therefore more expensive. The price of North intermediates additionally depends on the trade costs and the price of upstream inputs imported from South. Given the labor market clearing conditions $\int_{\tilde{s}}^{1} Q_N(s)ds = L_N$ and $\int_0^{\tilde{s}} Q_S(s)ds + Q_S^T = L_S$ as well as the price boundary conditions, the vertical specialization between North and South can be characterized as follows:

LEMMA 1 [Incomplete specialization]. There exists a unique cutoff $\tau_1 = \frac{a\lambda_N}{e^{\lambda_N}-1}$, below which North starts to source upstream stages to South. Before South fully specializes, the equilibrium is characterized by equations (9), (12) – (17):

$$w_S = \frac{1}{a}, \tag{12}$$

$$1 - \tilde{s} = -\frac{1}{\lambda_N}\ln(1 - \frac{\lambda_N L_N}{Q(\tilde{s})/\tau}), \tag{13}$$

$$Q(\tilde{s}) = e^{-\lambda_S \tilde{s}} Q_0, \tag{14}$$

$$p(\tilde{s}) = \frac{(e^{\lambda_S \tilde{s}} - 1)w_S}{\lambda_S}, \tag{15}$$

$$p(1) = e^{\lambda_N(1-\tilde{s})}\tau p(\tilde{s}) + (e^{\lambda_N(1-\tilde{s})} - 1)\frac{w_N}{\lambda_N} \equiv 1, \tag{16}$$

$$L_S^T = L_S - \frac{Q_0(1 - e^{-\lambda_S \tilde{s}})}{\lambda_S}, \tag{17}$$

with the regularity condition $L_S^T > 0$.

L_S^T denotes the amount of labor employed for traditional production in country S, $Q(\tilde{s})$ is the cutoff-stage output, and Q_0 is the output at stage zero. As there is only one value chain, I drop the country subscripts on price and quantity for simplicity.

With seven unknowns, $\tilde{s}$, w_N, w_S, $p(\tilde{s})$, $Q(\tilde{s})$, Q_0 and L_S^T, and seven equations (9, 12 – 17), the system is exactly identified. Equations (15) and (16) are directly derived from (10) and (11); equations (13), (14) and (17) are derived from equations (1), (2) and the market clearing conditions. Equation (13) suggests that the number of stages that North operates in depends positively on the country size and productivity. Equation (14) reflects the fact that intermediates get lost along stages. At the beginning of the production fragmentation when trade costs are not low enough, the sourced production stages cannot absorb the entire labor force in South. As such, traditional production coexists with modern production in South.

In the Appendix, I show that L_S^T increases in τ. Therefore, as τ decreases, the amount of labor employed by modern-production firms increases in South. Since from Costinot et al. (2013) we know that South is fully specialized under free trade, there must exist another cutoff value of trade costs below which all South workers are employed for modern production. In this new equilibrium, price and output equations (13) – (16) still hold. The only difference is the wage equation (12) being replaced

by the new labor market clearing condition in South:

$$L_S = \frac{Q_0(1 - e^{-\lambda_S \tilde{s}})}{\lambda_S}. \tag{18}$$

This gives the next lemma:

LEMMA 2 [Complete specialization]. There exists a unique cutoff τ_2 ($\tau_2 < \tau_1$), below which modern production employs all labor in South. After South fully specializes, the equilibrium condition is characterized by equations (9), (13) – (16) and (18).

The model describes three stages of development of South as trade costs in intermediates decrease, namely from no modern production to incomplete industrialization (modern production coexists with traditional production), and to complete industrialization. Admittedly stylized, the model matches well the development trajectory of countries like China or Thailand, who joined GVCs in the 1980s and achieved substantial economic growth with a massive move of labor from farms to factories.

Next I investigate how a decrease in trade costs affects production stages, real wages and the wage gap between North and South in both the complete and incomplete specialization equilibrium. The first comparative static results are as follows:

PROPOSITION 2. At incomplete specialization, as trade costs τ decline, South wage stays the same while North wage increases. The wage gap between North and South increases. Traditional production decreases in South, while modern production expands. South moves up the value chain, and world output of the final good increases.

I define that South "moves up" the value chain if the cutoff stage $\tilde{s}$ increases. The intuition behind proposition 2 is simple. Reductions in trade costs lead to further specialization between North and South; hence, world production of the final good increases. The increased final good production has to come from modern production, which implies that North, given its fixed amount of labor, must produce in fewer stages. As such, the cutoff stages $\tilde{s}$ must increase – which in turn implies more workers in South are employed for modern production. The real wage in South is pinned down by traditional production when it is incompletely specialized. Therefore, the gains from further specialization brought about by reduction in trade costs accrue entirely to North. As a result, North real wage increases and the wage gap between North and South widens.

PROPOSITION 3. At complete specialization, as trade costs τ fall, both countries' wages increase, and wage inequality decreases. South moves up the value chain, and world output of the final good increases.

Similar to the intuition behind proposition 2, a decrease in trade costs leads to efficiency gains and hence increases world production of the final good. This implies the cutoff stages $\tilde{s}$ must increase given the fixed labor supply in North. Since South is fully specialized, the real wage increases in both countries as trade costs fall. Changes in the world income distribution are more subtle. From equation

$(\lambda_N, \lambda_S) = (0.2, 0.8)$, $(L_N, L_S) = (1, 1)$, $a = 1.35$.

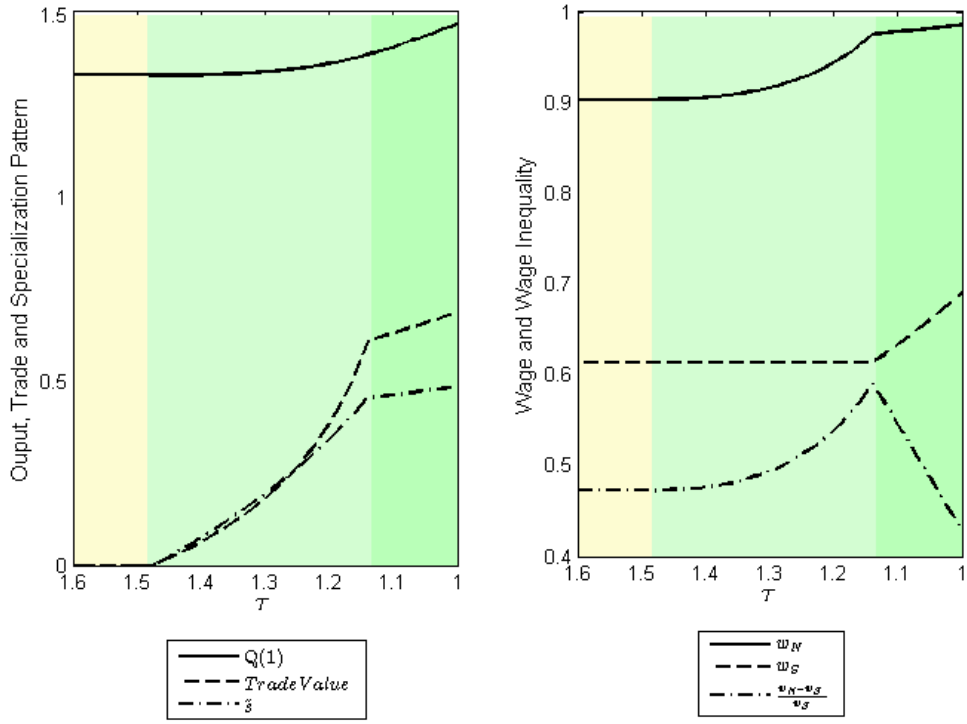

Figure 1: Consequences of decreases in trade costs

(9), we see that a decrease in τ is directly associated with a decrease in wage inequality. However, the decrease in τ also leads to an increase in $p(\tilde{s})$, which leads to a higher wage inequality indirectly. These two forces work in opposite directions, and the direct effect dominates in equilibrium.

An alternative way to understand proposition 3 is that, in complete specialization, the effect of trade costs on specialization is technically equivalent to the effect of labor-augmenting technology. Thus, the result of Costinot et al. (2013) on how countries' position in value chain changes with labor-augmenting technological progress directly applies in this context. However, a reduction in trade costs gives the opposite prediction regarding the world income distribution. This is because a decrease in τ directly lowers the import price of North, which increases the labor cost share of North exports, and in turn, reduces inequality.

Figure 1 illustrates how the pattern of vertical specialization, trade flows, world total production and wages respond to changes in trade costs in intermediates. The right panel of Figure 1 depicts the changes in wages in North and South, as well as the wage gap between North and South. The left panel of Figure 1 illustrates how trade, world output and cutoff stages vary with different trade costs. There are three distinct regions, from right to left, corresponding to the range of τ that yields separate production, incomplete specialization and complete specialization, respectively. When trade costs are very high, there is no production-sharing and a small decrease in τ does not change the production structure. When trade costs become sufficiently low, countries will move to incomplete specialization and finally to complete specialization if costs further decrease. In the beginning of the process, the

domestic wage of South does not rise because of the abundant labor in traditional production; the wage gap between North and South widens. As global unbundling continues, modern production employs all the labor force in South and its real wage starts to rise. In this period, a decrease in trade costs raises real wages in both countries, and the wage gap between North and South starts narrowing. Throughout the entire process, both trade flows and world final goods production increases as trade costs fall, and South moves up the value chain.

2.4 Discussions

Stylized facts and trade costs

The analysis provided in Section 2.3 confirms several stylized facts that have been documented in the literature. First, since a drop in trade costs implies that South moves up the value chain, we should observe an increased complexity in developing countries' production and exports. This has been documented by Malerba (1992), Levitt et al. (2013) and Thornton and Thompson (2001). Second, in my model, trade in value-added consists of all intermediate trade from South to North, plus North value-added embedded in final goods that is shipped from North to South. As there is additional value added in each stage, value-added trade should increase steadily as trade costs decline, as documented by Johnson and Noguera (2012a). Third, the model generates a non-monotonic real wage response to trade cost changes that matches the anecdotal evidence on the evolution of developing countries' real wages (Jaumotte and Tytell, 2007).[2] Finally, as shown in the left panel of Figure 1, when South is incompletely specialized, international trade grows much faster than world total output, which is consistent with the observation that the ratio of world trade to output has increased over recent decades (Hummels et al., 2001).

Policy reports such as the ones by OECD argue that trade frictions matter more in a world with GVCs (OECD, 2013), as intermediates often cross borders several times before a final good is made. My model provides additional insight on why trade costs matter more for GVC trade. The value-added nature of sequential production implies that the unit value of intermediate inputs increases along the value chain. Since the cutoff stage increases as trade costs decline, intermediates shipped across borders become more valuable. This implies that proportional trade costs become effectively more costly.

Transformed industrialization

The analysis provides a complementary explanation for the failure of the first and second generation of "Big Ideas" (Lindauer et al., 2002), and the mixed evidence for the success of "big push" development as documented by Bjorvatn and Coniglio (2012), Easterly (2006), Kline and Moretti (2014) and

[2] Jaumotte and Tytell (2007) found that since the 80s, in early Asian developers, such as Korea, Singapore, and Hong Kong SAR, manufacturing real wages have been converging rapidly toward U.S. levels. But wages in other Asian countries, including China, have been converging at a slower pace and only accelerated after 2000.

Magruder (2013), among others. The idea of simultaneous industrialization (i.e., "big push") was first introduced by Rosenstein-Rodan (1943), which was presented formally by Murphy et al. (1989), among others. This literature argues that if some sectors of an economy adopt increasing return to scale technologies simultaneously, they could create income that becomes a source of demand for goods in other sectors. Because of this externality, simultaneous industrialization of many sectors could be self-sustaining even if no sector could break even industrializing alone. However, Murphy et al. (1989) also point out that industrialization may be so costly that it reduces income and therefore the size of other firms' market, rendering no industrialization as the only stable equilibrium.

Similarly, in my model, when $\frac{1}{a} > \frac{\lambda_S}{e_S^{\lambda}-1}$, the scenario of complete and independent modern production in South will not emerge at any equilibrium, because firms always have the incentive to deviate and produce cheaper final goods using traditional production. Admittedly stylized, the broad message of this result is important: when modern production is not efficient due to bad institutions, poorly endowed human capital or other factors, organizing modern production can be costly and not necessarily welfare improving. In that case, "big push" strategies will fail to lead South towards prosperity.

Conversely, the reduction in trade costs can transform developing nations to world-class exporters by enabling them to specialize in parts of the modern production process. Without GVC trade, if South wants to develop a competitive modern industry base, it has to build production capacity for a broad range of intermediates. Such tasks are arduous, and only a few countries, like Japan and South Korea, have succeeded historically. Nevertheless, with the GVC trade, developing countries now have an alternative way to modernize, namely by producing intermediate goods for the advanced North. In this sense, my model offers a formal presentation of Baldwin (2011), Gereffi and Sturgeon (2013) and Foray (2014), on why joining GVCs matters for a county's industrialization strategy and how it differs from building value chains.

My analysis therefore points out a new industrialization possibility. However, it is important to note that, without productivity catch-ups, my model predicts that South always specializes at the upstream of the value chain and earns lower wages. In other words, although GVC trade made the "big push" industrialization strategy no longer necessary, technology improvements are still important.

3 Multi-country Extensions

Countries that are inside and outside of GVCs face distinct development questions. For outside countries, the most often asked question are: how can one successfully join the value chain? Is being geographically close to active GVC participants like China a blessing or a curse? For countries that are already part of the value chain, the most important questions probably are, how can they move up the value chain and achieve greater economic success? Moreover, are jobs created by GVC trade footloose? If so, would the engagement of Vietnam or Bangladesh, for instance, cause countries like China to lose their manufacturing jobs? To tackle these questions, I turn to one-North-multi-South extensions of the benchmark model in this section.

3.1 The General Specialization Patterns

To facilitate the analyses, I start by characterizing the vertical specialization in a general case. Consider J countries indexed by i, j, which have different productivities and face different bilateral trade costs. Triangular inequality $\tau_{ik}\tau_{kj} > \tau_{ij}$ always holds. I define $C^V \equiv \{i', ..., j'\}$ constitutes a value chain V, if there exists a final good whose production stages are located in those countries. In a multi-country setting where multiple value chains could coexist in equilibrium, the "global value chain" is more like a "global production network". Nevertheless, as shown in the Appendix, within each value chain, the vertical specialization discussed in Section 2 still holds. Ordering countries $i' \in C^V$ so that $\lambda_{i'}$ is weakly decreasing in c, the following corollary is immediate:

COROLLARY 1. Within any value chain, the allocation of stages to countries $C^V : (0, 1] \to C^V$ is an increasing function of s.

According to corollary 1, within any value chain, more productive countries always produce and export at more downstream stages of production. The proof of corollary 1 proceeds in two steps. First, by definition, if $C^V \equiv \{i', ..., j'\}$ constitutes a value chain V, any country $c \in C^V \equiv \{i', ..., j'\}$ must directly import or export intermediate inputs from at least one country inside of C^V. Second, as proved for proposition 1, if there is direct production-sharing between two countries, the intermediates must flow from the less productive to the more productive country. Therefore the allocation of stages to countries $C^V : (0, 1] \to C^V$ must be an increasing function of s.

COROLLARY 2. If trade frictions exist and $\lambda_i = \lambda_j$, there is no production-sharing between any i, j in equilibrium.

Corollary 2 can be proved by contradiction. From corollary 1, we know that, if two countries with the same technology operate in the same value chain, they must be ordered next to each other. Therefore, there is trade in intermediates between i and j. Since $\lambda_i = \lambda_j$, equation (9) requires $w_i = w_j\tau_{ij}$. However, if $w_i = w_j\tau_{ij}$, firms in country i are indifferent to using intermediates produced domestically or from country j at each stage that country i operates in. Therefore, the stage that country i operates in will also be operated in by country j. Denote the highest stage country i operates in as $\bar{s}$. If $\bar{s} = 1$, then the final good produced in country j will be cheaper, which contradicts the fact that the price of the final good is equal across nations. If $\bar{s} < 1$, then by the triangular inequality, the downstream consumer of country i will strictly prefer to source goods from country j instead. This contradicts the fact that country i produces at stage $\bar{s}$ and hence concludes the proof. The intuition behind corollary 2 is simple: with $\lambda_i = \lambda_j$, there are no gains from specialization and therefore no need for production-sharing if trade is costly.

3.2 Two Thought Experiments

With the presence of trade frictions in a multiple country setting, fully characterizing a many country equilibrium is a very difficult task. As the equilibrium is jointly determined by trade costs, technology

and the existing production networks, the number of possible equilibria explodes as the number of countries increases. Therefore, in this section I focus on a special one-North-multi-South case and repeat the thought experiment of Section 2. In this setting, South countries have the same productivity parameter λ_S but trade costs may vary. I am interested in the joining pattern of South nations as trade costs fall, and in understanding its impact on the welfare of countries that are already part of the production network. The set of countries is now defined as $C = \{N, S_1, \ldots, S_K\}$. Like in Section 3.1, I impose no assumptions on bilateral trade costs, except the triangular inequality. South countries have no modern production to start with. To characterize the change in *global* trade costs, I introduce a new parameter θ. A decrease in *global* trade costs is then defined as a change in bilateral trade frictions from $\tau_{cc'}$ to $\theta\tau_{cc'}$, with $\theta < 1$ for all $c, c' \in C$.[3]

Identical South countries and the hub-and-spoke economy

According to corollary 2, there will be no production-sharing between nations with the same productivity. Therefore, the global production network must be of a hub-and-spoke structure, with the headquarter economy, North, operating in downstream stages while factory South countries operate in the upstream stages. And every South provides intermediate inputs directly to North. Moreover, as shown in the proof of proposition 1 in the Appendix, if the cutoff stage between North and a South is $\tilde{s}$, then North will not produce in stages lower than $\tilde{s}$. Therefore, given the hub-and-spoke production structure, the cutoff stages need to be the same for all South countries that engage in GVC trade. The next proposition is therefore immediate:

PROPOSITION 4 [Specialization]. If international production-sharing exists, it must be that there exists a cutoff stage $\tilde{s} \in (0, 1)$ such that $Q_S(s) > 0$ if and only if $s \in (0, \tilde{s}]$ for all joined South nations, and $Q_N(s) > 0$ if and only if $s \in (\tilde{s}, 1]$.

Next, I characterize the joining pattern of South nations. Denote the set of South nations inside the value chain (inside South for short) by C_{S^I} and the jth South that joined by S_j^I, with superscript I indicating *"insiders."* Denote $C \setminus C'$ the relative complement of C' with respect to the set C. The joining sequence of South countries, as well as the impact of a South's joining on other value chain participants, can be summarized by the following proposition:

PROPOSITION 4 [Joining]. As θ decreases, the joining sequence of South countries is such that $S_j^I = \text{argmin}_{S_i \in C \setminus \{N\}}\{\tau_{NS_i}\}$ for $j = 1$; $S_j^I = \text{argmin}_{S_i \in C \setminus \{N, S_1^I, \ldots, S_{j-1}^I\}}\{\min_{k \in \{1,\ldots,j-1\}} \tau_{NS_k^I}\tau_{S_k^I S_i}\}$ for $j \geq 2$. The entry of a new South increases the real wage of North but decreases the real wage of other inside South countries. All South countries move up the value chain.

Proposition 4 [Joining] explains that the South that has the lowest trade cost with North will join the value chain first. Since South countries can only join a value chain from the most upstream stages, the second South will join when its trade cost with the first-joined South becomes

[3] θ plays the same role as τ in the paper, it is introduced for expositional purposes.

smaller than their wage gap. Then the third South will join when its trade cost with either the first- or the second-joined South (as they all produce the most upstream intermediates) becomes smaller than their respective wage gaps, and this process continues. As such, the jth successfully joined South must have the largest trade-costs-adjusted wage gap with one South country that is inside of the value chain. Since all inside South countries have the same productivity, their wage in turn depends only on their trade costs with North ($\tau_{NS_k^I}$). This is exactly the intuition behind $S_j^I = \text{argmin}_{S_i \in C \setminus \{N, S_1^I, \ldots, S_{j-1}^I\}} \{\min_{k \in \{1,\ldots,j-1\}} \tau_{NS_k^I} \tau_{S_k^I S_i}\}$ for $j \geq 2$. [4]

Proposition 4 suggests that except for the first-joined South, proximity (low τ) with joined South nations also matters for successful joining, even though there is no direct trade between joined South nations in equilibrium. Therefore the inside South countries, namely the "factory economies", are likely to be regionally clustered.

Having the same productivity, the joining of a new South can be thought of as an increase in labor supply for South countries that are inside the value chains. This lowers South countries' wages while raising the cutoff stage $\tilde{s}$, because the increased production implies that North has to specialize in fewer stages of production. This is an important result, as it implies that South occupying more production stages of a value chain does not necessarily improve its welfare. Moving up a value chain can be associated with welfare losses as well.

PROPOSITION 4 [Decreases in global trade costs]. Conditional on no new entry, if there exists an incompletely specialized South, a decrease in trade costs raises North wage but has no effect on South countries' wages. North-South inequality increases. If all countries are completely specialized, a decrease in trade costs leads to a real wage increase for all countries inside the production network, and North-South inequality decreases. In both cases, South countries move up the value chain, and the total output of the final good increases.

As proved in the Appendix, we can view all South countries as an aggregated economy whose size is a trade-costs weighted sum of all South countries. Therefore, the comparative static results of the two-country model directly apply.

The consequences of a decline in global trade frictions are illustrated in Figure 2. The left panel explains how the change in cutoff stages, trade flows and final output change with respect to θ; the right panel illustrates the response of wages and wage gaps. From left to right, the colored areas indicate the range of θ that corresponds respectively to the following scenarios: (1) separate production, (2) the first South joins the value chain but is incompletely specialized, (3) the first South completely specializes, (4) the second South joins but is incompletely specialized, and (5) both South countries are completely specialized. As trade costs fall and South countries join the value chain, the total output of the final good, trade in intermediates, and North wage increase. The wage of the South country that joined the value chain first increases, and then decreases when the second South joins. After the second South is completely specialized, both South countries' wages increase as θ falls.

[4] See the Appendix for the formal proof.

$(\lambda_N, \lambda_S) = (0.2, 0.92)$, $(L_N, L_{S_1}, L_{S_2}) = (1, 0.5, 1)$, $a = 1.62$, $(\tau_{NS_1}, \tau_{NS_2}, \tau_{S_1S_2}) = (1.111, 1.122, 1.12)$.

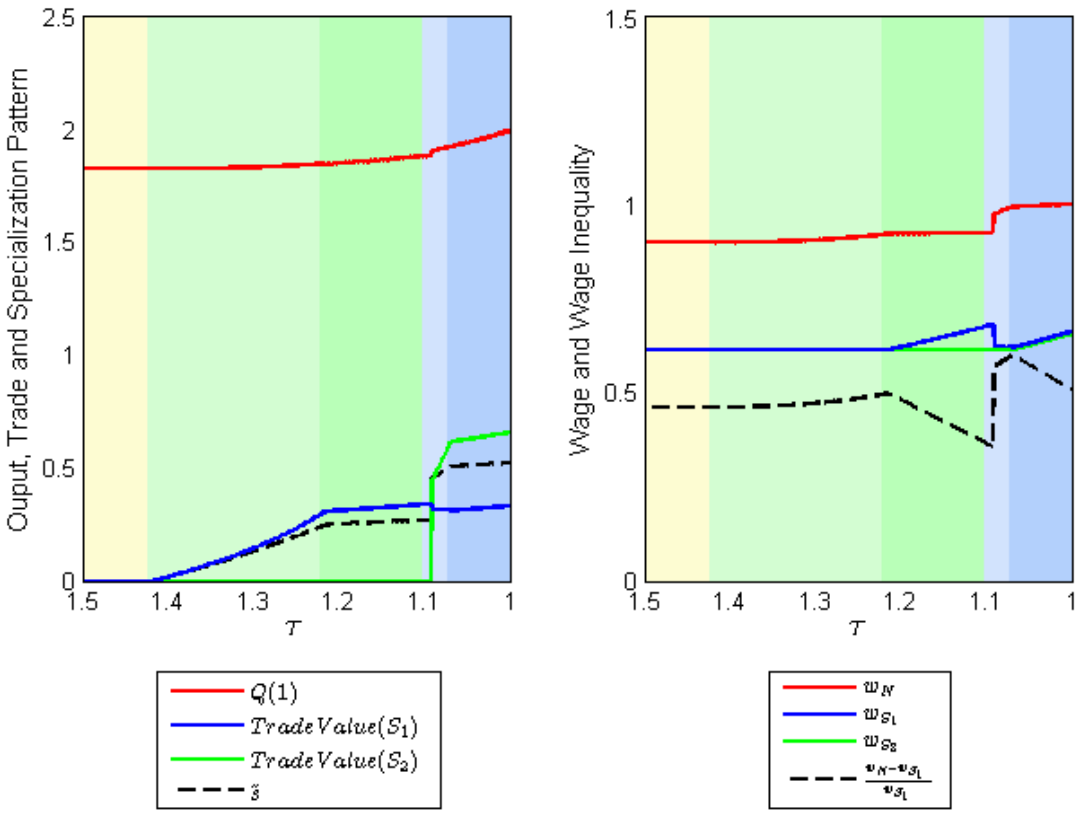

Figure 2: Consequences of decreases in trade costs (hub-and-spoke)

Learning-by-doing and the flying geese pattern of development

In the previous analyses, I assumed that all South nations have the same fixed technology λ_S. However, it is possible that South countries' productivity increase as their modern-production experience accumulates. Such a "learning-by-doing" mechanism has been documented by several empirical studies such as Thornton and Thompson (2001), Levitt et al. (2013) and Malerba (1992), among others.

To capture this idea, I propose a simple extension by assuming that $\lambda_{S_i} = q(t_i)\lambda_S$ for S_i, where t_i indicates how long S_i has been in the value chain and $q(.)$ is a monotonically-decreasing function with respect to t, with $q(.) \in (\lambda_N/\lambda_S, 1)$. In other words, I assume that a joined South will become more productive over time, but never become as productive as North or South countries that have joined before it.[5]

According to corollary 1, in equilibrium, countries are ordered based on their productivity along a value chain. Since I assume that South countries' productivity is strictly related to their time of joining, the last-joined South will always produce at the most upstream stages of the value chain. Therefore,

[5] I assume that no two nations have exactly the same trade frictions with a third nation; otherwise, there may be a possibility that two South nations join at the same time, resulting in a mix of hub-and-spoke and snake specialization.

$(\lambda_N, \lambda_{S_1}, \lambda_{S_2}) = (0.2, 0.5, 0.9)$ when S_2 joins, $(L_N, L_{S_1}, L_{S_2}) = (1, 0.5, 1)$, $a = 1.62$, $(\tau_{NS_1}, \tau_{NS_2}, \tau_{S_1S_2}) = (1.111, 1.122, 1.12)$.

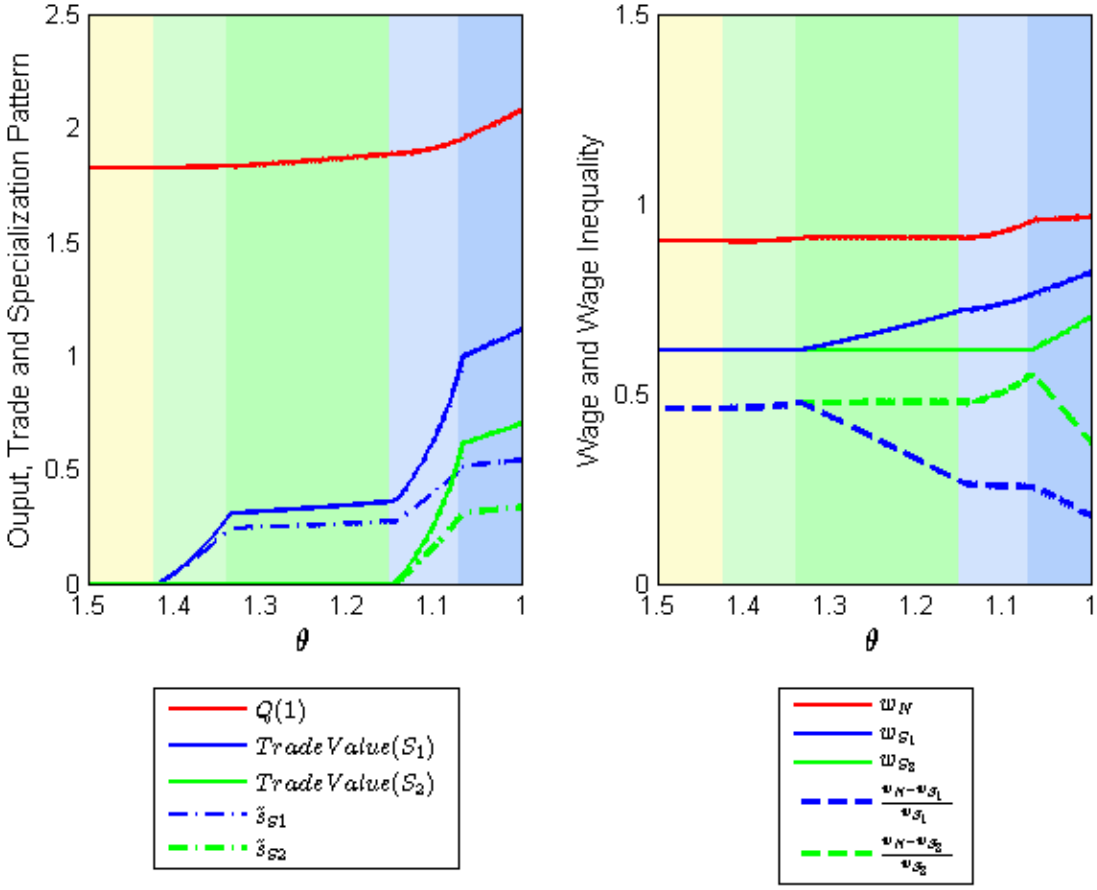

Figure 3: Consequences of decreases in trade costs (flying geese, strong learning effect)

international production has a sequential structure, with North operating at the most downstream stages and South countries producing at upstream stages following their joining sequence. The next proposition is therefore immediate:

PROPOSITION 5 [Specialization]. If international production-sharing exists, a sequence of stages, $0 \equiv s_0 < s_1 < ... < s_k < s_N = 1$ can be found such that $Q_N(s) > 0$ if and only if $s \in (s_k, 1]$ and $Q_{S_i^I}(s) > 0$ if and only if $s \in (s_{k-1}, s_{k-i+1}]$ for all $S_i^I \in C_{S^I}$.

When trade costs decrease, increased factor prices among inside countries provide opportunities for outside South countries because of their wage advantage. Since a non-industrialized South can join a value chain only from the most upstream stage and there is only one value chain, the next proposition is immediate:

PROPOSITION 5 [Joining]. As θ decreases, the joining pattern of South countries is such that $S_j^I = \text{argmin}_{S_i \in C \setminus \{N\}} \{\tau_{NS_i}\}$ for $j = 1$; $S_j^I = \text{argmin}_{S_i \in C \setminus \{N, S_1^I, \ldots, S_{j-1}^I\}} \{\tau_{S_{j-1}^I S_i}\}$ for $j \geq 2$.

Proposition 5 [Joining] explains that the South that has the lowest trade cost with North will join the value chain first; the joining sequence of other South nations depends on their proximity to

$(\lambda_N, \lambda_{S_1}, \lambda_{S_2}) = (0.2, 0.5, 0.51)$ when S_2 joins, $(L_N, L_{S_1}, L_{S_2}) = (1, 0.5, 1)$, $a = 1.62$, $(\tau_{NS_1}, \tau_{NS_2}, \tau_{S_1S_2}) = (1.111, 1.122, 1.12)$.

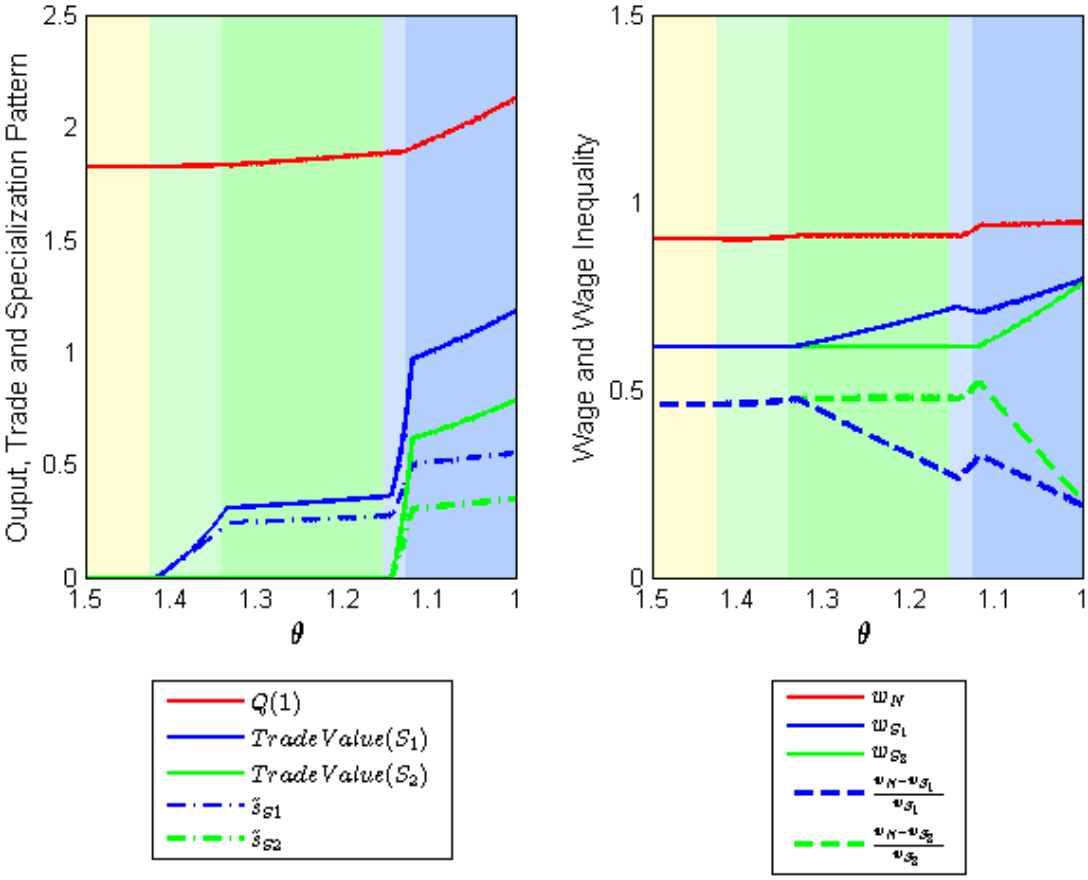

Figure 4: Consequences of decreases in trade costs (flying geese, weak learning effect)

the latest-joined South. Compared to the case of a hub-and-spoke economy, sequential specialization provides a stronger version of the regional clustering of factory economies, as only the proximity to the latest-joined South matters. If trade frictions are positively correlated with distance, the model implies that the developing countries that join sequentially are likely to be geographically close to each other. This implies that being close to countries like China is a blessing to nearby countries that try to participate in GVC trade. Another way to read proposition 5 is that developing countries in the same region are likely to embrace GVC trade sequentially. This sequential regional development pattern is reminiscent of Akamatsu's flying geese pattern of development (Akamatsu, 1962; Kojima, 2000), which explains that a less-advanced country's economy could develop by "entering into an international economic relationship with the advanced countries" (Akamatsu, 1962).

When international specialization has a sequential structure, the impact of decreases in trade costs can be summarized by the following proposition:

PROPOSITION 5 [Decreases in global trade costs]. The global decrease in trade costs increases the total production of the final good and allows all countries inside the value chain to move up. When countries are fully specialized, a decrease in trade costs leads to an increase in real wages and a

decrease in wage inequality for all countries. When the newly-joined South is incompletely specialized, a decrease in trade costs raises North wage but has an ambiguous impact on the wage of other inside South countries.

The intuition behind the change in total production and countries' specialization stages is akin to that behind propositions 2 and 3. A fall in trade costs enables further specialization between countries; hence, the final good production increases. This means the most downstream country has to produce more intermediate inputs for each stage it operates in. As a result, the country has to produce in fewer stages given its fixed labor supply. Furthermore, its decrease in production stages must be offset by an expansion in stages occupied by other countries. Proceeding by iteration from the most downstream of the value chain, one can show that this can occur only if all countries move up.

When all countries are specialized, a drop in trade costs raises the real wage for all countries as the final good becomes cheaper. Trade frictions act as a wedge to differentiate wages across countries, so income inequality decreases when trade costs fall. The wage response for the incomplete specialization case is more subtle. Recall in equation (9), a country's wage depends on trade frictions, its upstream country's wage, cutoff stage prices and the efficiency gap between the two countries. If a country close to the most upstream is only slightly more efficient than the newly-joined South, the second term of equation (9) will be close to zero. As such, the overall effect will be dominated by the lower trade frictions, which narrows the wage gap between the country and the newly-joined South, whose wage is pinned down at $\frac{1}{a}$. In this case, countries that are close to the most upstream stages of the value chain are likely to experience a decrease in their wages.

The consequences of decreasing trade costs with learning-by-doing are illustrated in Figure 3 (strong learning effects) and Figure 4 (weak learning effects). The left panel of Figure 3 explains that, as trade costs fall, the first South joins the value chain and expands its production range. After its wage increases to a certain point, the second South joins and repeats the pattern. As one can see from the right panel of Figure 4, when technology gap is small between South nations (i.e. efficiency gains from learning-by-doing are insignificant), an inside South is likely to experience a real wage decline when a new South joins. When learning-by-doing causes significant productivity spillovers, the story is the opposite (the right panel of Figure 3).

3.3 Discussions

The regional nature of GVC trade

An well-known feature of GVC trade is that it is regional rather than global (Baldwin and Lopez-Gonzalez, 2015; Johnson and Noguera, 2012b). For instance, the North American auto industry is more geographically dispersed than it was in the 1950s, but many auto-parts plants are still clustered within a 1,000-kilometer radius from Michigan, with major outsourcing firms located in Mexico (Klier and Rubenstein, 2008). Likewise, most factories that Japan and South Korea firms source from are located in nearby countries like China, Vietnam and Thailand (Baldwin, 2008). The extensions

presented in this section capture this feature. Particularly, in the flying geese case, the success of a South's entry into a value chain depends solely on its proximity to the factory economy at the most upstream stage of the value chain. This result is important as it provides insights for countries that aim to develop by joining GVCs: reducing trade frictions with the developed North might not be enough. Instead, rethinking where and how to fit into GVCs requires an integrated analysis of the production network and the country's proximity and technology difference with all inside countries.

New norm of the flying geese pattern of development

As discussed briefly in subsection 3.3, introducing a simple learning-by-doing mechanism creates a new flying geese pattern of development. However, this is quite different from what Akamatsu originally had in mind. Written in the 1960s, Akamatsu's initial idea is classic: a developing country can achieve industrialization by first performing import-substitution, then producing goods independently for domestic use, and finally exporting. In contrast, my model sheds light on how the second unbundling has transformed the industrialization possibilities for developing countries. Akamatsu's three-stage development now becomes the following: in the first stage, South industrializes by joining GVCs and performs a narrow set of production stages. Workers are increasingly employed for modern production, but wage remains low. In the second stage, the range of intermediates produced in South expands and its wage increases rapidly. In the last stage, the increased wage in this South country attracts other developing countries to join, and the South re-specializes at higher stages of the value chain and produces a narrower range of, more complicated intermediates. Throughout this process, the country is always a part of a global production network, and its welfare is closely linked with the development of other GVC participants.

4 Policy Implications

The international fragmentation of production brings new policy challenges. My analyses indicate that the policy implications of GVC participation depend on a country's development level, the existing international production structure and the potential technology spillovers. In this section, I briefly discuss some policy concerns regarding GVC trade and the importance of multilateral GVC governance.

There are three group of countries facing very different challenges. The first is developing nations that aim at joining GVCs. My model indicates that, for those nations, what matters is not only their proximity to the advanced North, but also their proximity to other nations at the very upstream of the value chain. For those nations, unilaterally reducing trade frictions to all nations seems the best way to facilitate joining, especially given the complex nature of international production networks in reality.

The second group is developing countries that are already participants of global production networks, but want to move up the value chain. My model indicates that facilitating trade with either relatively less efficient or more efficient countries is helpful. However, reducing trade costs with outside

countries that have similar technologies may decrease their welfare. This can be clearly seen from both the hub-and-spoke and the flying geese analyses. In both cases, an inside South will have the incentive to impede other South countries in joining the value chain. For those countries, the optimal trade policy may involve facilitating trade with some countries but impeding it with others, depending on their productivity and positions inside the global production networks.

The last set of countries are advanced economies like Japan, South Korea and the United States, which were already industrialized before the second unbundling and now face a "hollowing out" of their industry base. My analyses indicate that this is not a concern in the long run. It may simply reflect that developed nations are specializing in more complex stages of the value chain, which is welfare-improving. For those economies, the key is to keep their technology leadership rather than prohibit offshoring.

The conflicted interest among developing countries inside and outside the value chain naturally leads to concerns regarding GVC governance. Bilateral trade agreements between North and South countries are of mutual interest and hence will be signed without problems. Regional agreements signed simultaneously by North and multiple South are also without problems. Challenges appear when inside and outside South countries have similar development levels. In this case, an established value chain may become a "stumbling block" for the further formation of global or regional production networks.

5 Conclusion

In this paper, I develop a model with sequential production and international trade frictions that permits an analysis of how a decrease in trade costs shapes the interdependence between countries. I show that, as trade costs fall, South joins and moves up the value chain. Both North and South gain from decreased trade costs, though the process is non-monotonic. In multi-country extensions, I show that countries are strictly ordered along stages by their productivity within each value chain, though the specialization pattern of countries inside a production network can be complex. The successful joining of new countries into the value chain depends on their proximity to insiders, as well as their initial industry base. In particular, in a one-North-multi-South setting, when global trade costs decrease, South countries form value chains with North sequentially. If they are of the same productivity, the production structure exhibits a hub-and-spoke shape, and the joining of the new South dampens the welfare of other South countries that are already inside of the production network. If they are of different productivities because of, say, learning-by-doing, the joining and development of South countries will exhibit a flying geese pattern. If the learning-by-doing effect is strong enough, the newly-joined South will benefit everyone inside the value chain. Factory economies are likely to be regionally clustered in both cases. This framework can be used to analyze policy questions raised by GVC trade.

Even though the model presented in this paper is stylized, it matches empirical regularities on GVC

trade and provides new insights into old development puzzles and recent GVC debates. Moreover, the model can be extended to answer other interesting questions without much complication. For instance, it provides a good framework to discuss GVC-governance-related questions, such as the "building" and "stumbling" nature of regional trade agreements, the optimal trade policy for economic development. It would also be interesting to extend the model to multi-good and multi-factor settings to analyze questions related to GVC trade and factor-price adjustment. Last but not least, introducing endogenous innovation and strategic trade policy to see how developing economies could achieve long-run growth by joining GVCs is an interesting and important area for further research.

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Appendix A Appendix

Proof of proposition 1. I prove proposition 1 in four steps.

Step 1. Price is strictly increasing along stages.

From equilibrium condition (3) we know that:

$$p_c(s) = p_{c'}(s - ds)(1 + \lambda_c ds) + w_c ds \text{ for } c = c', \forall s, \tag{A1}$$

$$p_c(s) = \tau p_{c'}(s - ds)(1 + \lambda_c ds) + w_c ds \text{ for } c \neq c', \forall s. \tag{A2}$$

In either case the intermediate goods price is strictly increasing along stages.

Step 2. With the presence of trade frictions, it would not be feasible for firms in country c both to source from and produce for firms in country c'.

If firms in country c find it optimal to produce at stage s by sourcing stage $s - ds$ goods from country c' and then export, it must be that downstream firms in country c' find it cheaper to purchase intermediates from them, rather than source domestically. This formally implies:

$$\tau(\tau p_{c'}(s - ds)(1 + \lambda_c ds) + w_c ds) \leq p_{c'}(s - ds)(1 + \lambda_{c'} ds) + w_{c'} ds,$$

rearranging terms we get:

$$p_{c'}(s - ds)(\tau^2 - 1) \leq [p_{c'}(s - ds)(\lambda_{c'} - \lambda_c \tau^2) + (w_{c'} - \tau w_c)]ds.$$

As long as $\tau > 1$, the above inequality will not hold as ds is infinitely small. This result shares the same intuition with Baldwin and Venables (2013), who argue that fragments below a minimum size will not be offshored.

Step 3. In any equilibrium with production-sharing, it must be that South firms supply upstream intermediates to North.

I prove this by contradiction. Suppose North firms produce at stage s and serve South firms at stage $s + ds$. From the previous result, we know that North must source domestically while South firms supply domestically. Then, equilibrium condition (3) implies that for this production-sharing structure to hold, we must have

$$\tau p_N(s - ds)(1 + \lambda_S ds) + w_S ds \geq \tau p_N(s - ds)(1 + \lambda_N ds) + \tau w_N ds, \tag{A3}$$

$$\tau p_N(s)(1 + \lambda_N ds) + \tau w_N ds \geq \tau p_N(s)(1 + \lambda_S ds) + w_S ds. \tag{A4}$$

Equation (A3) and (A4) together imply:

$$p_N(s - ds) \geq (\tau w_N - w_S)/\tau(\lambda_S - \lambda_N) \geq p_N(s), \tag{A5}$$

which contradicts the strictly increasing prices.

Step 4. If South firms supply upstream intermediates to North at stage s, then North cannot produce at $s' \leq s$.

I prove this by contradiction. Suppose North sources intermediate inputs from South at $\tilde{s}$, Price condition (3) again requires:

$$\tau p_S(\tilde{s} - ds)(1 + \lambda_N ds) + w_N ds \geq \tau p_S(\tilde{s} - ds)(1 + \lambda_S ds) + \tau w_S ds,$$

and

$$\tau p_S(\tilde{s})(1 + \lambda_S ds) + \tau w_S ds \geq \tau p_S(\tilde{s})(1 + \lambda_N ds) + w_N ds,$$

which together imply:

$$w_N = \tau w_S + \tau p(\tilde{s})(\lambda_s - \lambda_N), \tag{A6}$$

since ds is infinitesimal.

Since $p(s)$ is strictly increasing in s, when production-sharing exists between North and South, there is at most one cutoff stage $\tilde{s}$. Otherwise we get two wage equations. Suppose North also produce at $s' \leq \tilde{s}$, then North must produce in all stages $s \in (0, 1]$ as there is maximum one cutoff stage. Results of step 3 imply that South firms produce at stage $s \in (0, \tilde{s}]$, as its firms producing at stage $\tilde{s}$ have to source domestically. Denote the prices firms in North and South charge at stage s' by $p_S(s')$ and $p_N(s')$, respectively. Goods market clearing then requires:

$$\tau p_S(s') \geq p_N(s'), \text{for } s' < \tilde{s}, \tag{A7}$$

otherwise North firms will not be able to produce at stage $s' < \tilde{s}$. Use the equation (3), we can solve intermediate prices in each country:

$$p_N(s) = (e^{\lambda_N s} - 1)\frac{w_N}{\lambda_N}, \forall s; \tag{A8}$$

and

$$p_S(s) = (e^{\lambda_S s} - 1)\frac{w_S}{\lambda_S}, s \in (0, \tilde{s}]. \tag{A9}$$

Combining equation (A8) and (A9), it is straightforward to show that $\frac{p_S(s)}{p_N(s)}$ is an increasing function of s. Since at the cutoff stage $\tilde{s}$ we have $\frac{p_S(\tilde{s})}{p_N(\tilde{s})} = \frac{1}{\tau}$, $\frac{p_S(s')}{p_N(s')}$ must be smaller than $\frac{1}{\tau}$ for $s' < \tilde{s}$. This contradicts the inequality (A7) and concludes the proof.

Proof of lemma 1.

Step 1. Separate-production wages and the cutoff τ_1.

I first derive wages of North and South before production-sharing. South has traditional production only, hence $w_S = \frac{1}{a}$. North has already "industrialized" and thus engages in modern production. By

price condition (3), the following price equation must hold:

$$p_N(s) = p_N(s - ds)(1 + \lambda_N ds) + w_N ds, \forall s, \tag{A10}$$

which implies

$$\frac{dp_N(s)}{ds} = \lambda_N p(s) + w_N, \forall s. \tag{A11}$$

Together with the boundary conditions $p(0) = 0$ and $p(1) = 1$, one can solve the previous differential equation:

$$w_N = \frac{\lambda_N}{e^{\lambda_N} - 1}. \tag{A12}$$

The regularity condition $\frac{1}{a} < \frac{\lambda_N}{e^{\lambda_N}-1}$ implies that w_S is smaller than w_N when South and North produce goods separately. Because $p(0) = 0$, when $\tau \leq \frac{a\lambda_N}{e^{\lambda_N}-1}$, North starts to source intermediates from South (starting from stage $(0, \Delta]$).

Step 2. Equilibrium conditions.

Before South fully specializes, its wage is still $\frac{1}{a}$. This gives us equation (12). Equation (9) is already derived from step 4 of the proof of proposition 1. Next let us consider equations (15) and (16). Proposition 1 implies that the price difference equation can be written as:

$$\frac{dp(s)}{ds} = \lambda_S p(s) + w_S, \forall s \in (0, \tilde{s}], \tag{A13}$$

and

$$\frac{dp(s)}{ds} = \lambda_N p(s) + w_N, \forall s \in (\tilde{s}, 1], \tag{A14}$$

with $p(\tilde{s} + ds) = \tau(1 + \lambda_N)p(\tilde{s}) + w_N ds$. Using the regularity condition $p(0) = 0$ to solve equation (A13), we get equation (15); using $p(1) = 1$ to solve equation (A14), we get equation (16).

I then turn to derive equations (13), (14), and (17). Proposition 1 and equation (1) imply:

$$\frac{dQ_S(s)}{ds} = \lambda_S Q(s), \forall s \in (0, \tilde{s}]. \tag{A15}$$

Denoting the total input at stage 0 as Q_0 and solving the previous difference equation we get equation (14). Then, using equation (2) and

$$\frac{dQ_N(s)}{ds} = \lambda_N Q(s), \forall s \in (\tilde{s}, 1], \tag{A16}$$

we get

$$Q(s) = \frac{1}{\tau} e^{-\lambda_N (s - \tilde{s})} Q(\tilde{s}), \forall s \in (\tilde{s}, 1]. \tag{A17}$$

Combining equation (A17) and the labor market clearing condition $\int_{\tilde{s}}^{1} Q(s) = L_N$, we get equation (13). Labor employed for modern production in South equals $\int_0^{\tilde{s}} Q(s)$; substituting $Q_S(s) = e^{-\lambda_S(s)} Q_0$

into this equation and collecting terms, we find that the labor employed for modern production equals $\frac{Q_0(1-e^{\lambda_S \tilde{s}})}{\lambda_S}$. The remaining labor is employed therefore for traditional production, which gives us equation (17).

Proof of lemma 2.

When South is fully specialized, its labor market clearing implies $L_s = \frac{Q_0(1-e^{\lambda_S \tilde{s}})}{\lambda_S}$ and $w_S = \frac{1}{a}$ no longer holds. It is straightforward to show that the remaining equations continue to hold in this equilibrium.

Proof of proposition 2.

I decompose the proof of proposition 2 into two steps. I first show that a decrease in τ leads to an increase of wages in North, and then I show that a decrease in τ increases the cutoff stage $\tilde{s}$ and the total amount of labor employed by the modern sector in South.

Step 1. A decrease in τ leads to an increase of w_N.

Equation (9) can be rewritten as:

$$p(\tilde{s}) = \frac{w_N - \tau w_S}{\tau(\lambda_S - \lambda_N)}. \tag{A18}$$

Plugging the previous equation into equation (15) to substitute $p(\tilde{s})$, and writing $\tilde{s}$ as a function of w_N and τ, I get:

$$\tilde{s} = \frac{\ln\left(\frac{\lambda_S(w_N - \tau w_S)}{\tau w_S(\lambda_S - \lambda_N)} + 1\right)}{\lambda_S}. \tag{A19}$$

Combining equations (A18) and (A19) to substitute $p(\tilde{s})$ and $\tilde{s}$ in equation (16), I get

$$e^{\lambda_N\left(1-\frac{\ln\left(\frac{\lambda_S(w_N-\tau w_S)}{\tau w_S(\lambda_S-\lambda_N)}+1\right)}{\lambda_S}\right)}\frac{w_N - \tau w_S}{\lambda_S - \lambda_N} + \left(e^{\lambda_N\left(1-\frac{\ln\left(\frac{\lambda_S(w_N-\tau w_S)}{\tau w_S(\lambda_S-\lambda_N)}+1\right)}{\lambda_S}\right)} - 1\right)\frac{w_N}{\lambda_N} = 1. \tag{A20}$$

Total differentiating equation (A20), I get:

$$\begin{aligned}&\left(\frac{\lambda_S e^{\lambda_N(1-\tilde{s})}}{(\lambda_S-\lambda_N)\lambda_N} - \frac{1}{\lambda_N} - \frac{e^{\lambda_N(1-\tilde{s})}(\lambda_S w_N - \tau\lambda_N w_S)}{\lambda_S-\lambda_N}\frac{\partial\tilde{s}}{\partial w_N}\right)dw_N \\ &-e^{\lambda_N(1-\tilde{s})}\left(\frac{w_S}{\lambda_S-\lambda_N} + \frac{w_N-\tau w_S}{\lambda_S-\lambda_N}\lambda_N\frac{\partial\tilde{s}}{\partial\tau} + \frac{w_N}{\lambda_N}\lambda_N\frac{\partial\tilde{s}}{\partial\tau}\right)d\tau = 0.\end{aligned} \tag{A21}$$

Taking the derivative of equation (A19) with respect to τ, I get:

$$\frac{\partial\tilde{s}}{\partial\tau} = -\frac{w_N}{\tau(\lambda_S w_N - \tau\lambda_N w_S)} < 0. \tag{A22}$$

Similarly, taking the derivative of equation (A19) with respect to w_N, I get:

$$\frac{\partial\tilde{s}}{\partial w_N} = \frac{1}{\lambda_S w_N - \tau\lambda_N w_S} > 0. \tag{A23}$$

Substituting equations (A22) and (A23) into equation (A21) and collecting terms, I get:

$$\left(\frac{e^{\lambda_N(1-\tilde{s})}}{\lambda_N} - \frac{1}{\lambda_N}\right) dw_N = \frac{e^{\lambda_N(1-\tilde{s})}}{\lambda_S - \lambda_N}\left(w_S - \frac{w_N}{\tau}\right) d\tau, \tag{A24}$$

where $\left(\frac{e^{\lambda_N(1-\tilde{s})}}{\lambda_N} - \frac{1}{\lambda_N}\right) > 0$. By equation (9) and $p(s) > 0$, it must be that $w_N - \tau w_S > 0$. Combining this inequality and $\lambda_S > \lambda_N$, one can verify that $\frac{e^{\lambda_N(1-\tilde{s})}}{\lambda_S - \lambda_N}\left(w_S - \frac{w_N}{\tau}\right) < 0$. Therefore $\frac{\partial w_N}{\partial \tau} < 0$.

Step 2. A decrease in τ leads to an increase in $\tilde{s}$ and a decrease in L_S^T.

I prove this by contradiction. Suppose for $\tau' < \tau$, we have the cutoff stage $\tilde{s}' \leq \tilde{s}$. Since there is no wage change in South because of the traditional sector, equation (15) implies $p(\tilde{s}') \leq p(\tilde{s})$. However, if this is the case, equation (9) implies $w'_N < w_N$, which contradicts the result of step 1. Therefore, it must be that $\tilde{s}' > \tilde{s}$.

Combining equations (13), (14) and (17), I get:

$$L_S^T = L_S - \frac{\tau \lambda_N L_N}{1 - e^{-\lambda_N(1-\tilde{s})}} \frac{e^{\lambda_S \tilde{s}} - 1}{\lambda_S}. \tag{A25}$$

Taking the derivative of the previous equation with respect to τ, I get:

$$\frac{\partial L_S^T}{\partial \tau} = -\frac{\lambda_N L_N}{1 - e^{-\lambda_N(1-\tilde{s})}} \frac{e^{\lambda_S \tilde{s}} - 1}{\lambda_S}\left(1 - \frac{w_N}{\lambda_S w_N - \tau \lambda_N w_S}\left(\frac{\lambda_S e^{\lambda_S \tilde{s}}}{e^{\lambda_S \tilde{s}} - 1} + \frac{\lambda_N e^{-\lambda_N(1-\tilde{s})}}{1 - e^{-\lambda_N(1-\tilde{s})}}\right)\right). \tag{A26}$$

It is straightforward to verify that $\frac{w_N}{\lambda_S w_N - \tau \lambda_N w_S} > \frac{1}{\lambda_S}$, $\frac{\lambda_S e^{\lambda_S \tilde{s}}}{e^{\lambda_S \tilde{s}} - 1} > \lambda_S$ and $\frac{\lambda_N e^{-\lambda_N(1-\tilde{s})}}{1 - e^{-\lambda_N(1-\tilde{s})}} > 0$, therefore the last term of equation (A26), $1 - \frac{w_N}{\lambda_S w_N - \tau \lambda_N w_S}\left(\frac{\lambda_S e^{\lambda_S \tilde{s}}}{e^{\lambda_S \tilde{s}} - 1} + \frac{\lambda_N e^{-\lambda_N(1-\tilde{s})}}{1 - e^{-\lambda_N(1-\tilde{s})}}\right)$, is positive. Thus $\frac{\partial L_S^T}{\partial \tau} > 0$.

Proof of proposition 3.

I decompose the proof of proposition 3 into four steps. I first show that a decrease in τ leads to an increase in final output and South moving up the value chain. Then, I show that it also leads to an increase of wages in both South and North. In the last step, I show that wage inequality between North and South decreases.

Step 1. A decrease in τ leads to an increase of $\tilde{s}$ and $Q(1)$.

I first consider the cutoff stage $\tilde{s}$. From equation (18) I know that $\frac{\partial Q_0}{\partial \tilde{s}} < 0$. Using equations (18) and (14) to eliminate $\tilde{s}$, I get:

$$Q(\tilde{s}) = Q_0 - \lambda_S L_S. \tag{A27}$$

Equations (18), (13), and (A27) imply

$$-\frac{1}{\lambda_N} \ln\left(1 - \frac{\tau \lambda_N L_N}{(Q_0 - \lambda_S L_S)}\right) - \frac{1}{\lambda_S} \ln\left(1 - \frac{\lambda_S L_S}{Q_0}\right) = 1. \tag{A28}$$

From the above equation it is easy to verify that $\frac{\partial Q_0}{\partial \tau} > 0$. Then equation (18) implies that if Q_0

decreases as τ decreases, $\tilde{s}$ must increase.

Now I examine how $Q(1)$ changes when τ decreases. Note that the final good production can be written as:

$$Q(1) = \frac{Q_0 e^{-(\lambda_S \tilde{s} + \lambda_N (1-\tilde{s}))}}{\tau}. \tag{A29}$$

Combining with equation (18), I obtain:

$$Q(1) = \frac{\lambda_S L_S}{\tau} \frac{e^{-\lambda_N (1-\tilde{s})}}{e^{\lambda_S \tilde{s}} - 1}. \tag{A30}$$

Next, I use equation (18) to eliminate Q_0 in equation (A28) to get:

$$-\frac{1}{\lambda_N} \ln\left(1 - \frac{\lambda_N L_N (1 - e^{-\lambda_S \tilde{s}})\tau}{\lambda_S L_S e^{-\lambda_S \tilde{s}}}\right) + \tilde{s} = 1, \tag{A31}$$

which provide a direct relationship between $\tilde{s}$ and τ:

$$\tau = \frac{\lambda_S L_S}{\lambda_N L_N} \frac{(1 - e^{-\lambda_N (1-\tilde{s})}) e^{-\lambda_S \tilde{s}}}{1 - e^{-\lambda_S \tilde{s}}}. \tag{A32}$$

Totally differentiating equation (A32), I get:

$$d\tilde{s} = \frac{1}{\alpha} d\tau, \tag{A33}$$

where $\alpha \equiv \frac{\lambda_S L_S}{\lambda_N L_N} \frac{-\lambda_N e^{-\lambda_N (1-\tilde{s})} e^{-\lambda_S \tilde{s}} (1 - e^{-\lambda_S \tilde{s}}) - \lambda_S (1 - e^{-\lambda_N (1-\tilde{s})}) e^{-\lambda_S \tilde{s}}}{(1 - e^{-\lambda_S \tilde{s}})^2}$. Clearly $\alpha < 0$. Totally differentiating equation (A30) I get:

$$dQ(1) = -\frac{Q(1)}{\tau} d\tau + Q(1) \left((\lambda_N - \lambda_S) - \frac{\lambda_S}{e^{\lambda_S \tilde{s}} - 1}\right) d\tilde{s}. \tag{A34}$$

Substituting equation (A33) into (A34), and then using equation (A32) to substitute $\frac{\lambda_S L_S}{\lambda_N L_N}$ in the new equation, I get:

$$\frac{dQ(1)}{Q(1)} = -\frac{1}{\tau} \left(1 - \frac{(1 - e^{-\lambda_S \tilde{s}})(1 - e^{-\lambda_N (1-\tilde{s})}) \left((\lambda_N - \lambda_S) - \frac{\lambda_S}{e^{\lambda_S \tilde{s}} - 1}\right)}{\lambda_N e^{-\lambda_N (1-\tilde{s})} (1 - e^{-\lambda_S \tilde{s}}) + \lambda_S (1 - e^{-\lambda_N (1-\tilde{s})})}\right) d\tau. \tag{A35}$$

Therefore showing $\frac{\partial Q(1)}{\partial \tau} < 0$ is equivalent to showing that :

$$(1 - e^{-\lambda_S \tilde{s}})(1 - e^{-\lambda_N (1-\tilde{s})}) \left((\lambda_N - \lambda_S) - \frac{\lambda_S}{e^{\lambda_S \tilde{s}} - 1}\right) < \lambda_N e^{-\lambda_N (1-\tilde{s})} (1 - e^{-\lambda_S \tilde{s}}) + \lambda_S (1 - e^{-\lambda_N (1-\tilde{s})}), \tag{A36}$$

which can be easily verified.

Step 2. A decrease in τ leads to an increase in both w_S and w_N.

I first write w_N as a function of $p(\tilde{s})$ and w_S using equation (9) and substitute it into the price

equation (16). This enables me to write $p(\tilde{s})$ as a function of w_S. Substituting it into equation (15) and collecting terms, I get:

$$\frac{\tau w_S}{\lambda_S \lambda_N}\left((e^{\lambda_S \tilde{s}}-1)\left(\lambda_S(e^{\lambda_N(1-\tilde{s})}-1)+\lambda_N\right)+\lambda_S(e^{\lambda_N(1-\tilde{s})}-1)\right)=1. \tag{A37}$$

Then, using equations (A37) and (A32) to substitute τ, I get:

$$\frac{L_S}{\lambda_N^2 L_N} w_S \psi = 1, \tag{A38}$$

where $\psi \equiv \frac{1-e^{-\lambda_N(1-\tilde{s})}}{e^{\lambda_S \tilde{s}}-1}\left((e^{\lambda_S \tilde{s}}-1)\left(\lambda_S(e^{\lambda_N(1-\tilde{s})}-1)+\lambda_N\right)+\lambda_S(e^{\lambda_N(1-\tilde{s})}-1)\right)$. With some algebra one can show that $\frac{\partial \psi}{\partial \tilde{s}}<0$, and thus $\frac{\partial w_S}{\partial \tilde{s}}>0$. As in step 1, I already showed that $\tilde{s}$ increases when τ decreases, therefore w_S also increases when τ decreases.

Similarly, one can write the final good price equation as a function of w_N and $\tilde{s}$ as follows:

$$\frac{w_N}{\lambda_S \lambda_N}\varphi = 1, \tag{A39}$$

where $\varphi \equiv \left(1+\frac{\lambda_S-\lambda_N}{\lambda_S}(e^{\lambda_S \tilde{s}}-1)\right)^{-1}\left((e^{\lambda_S \tilde{s}}-1)\left(\lambda_S(e^{\lambda_N(1-\tilde{s})}-1)+\lambda_N\right)+\lambda_S(e^{\lambda_N(1-\tilde{s})}-1)\right)$. After some algebra one can show that $\frac{\partial \varphi}{\partial \tilde{s}}<0$. Thus, $\frac{\partial w_N}{\partial \tilde{s}}>0$. As in step 1, $\tilde{s}$ increases when τ decreases, therefore w_N also increases when τ decreases.

Step 3. A decrease in τ leads to a decrease in wage inequality $\frac{w_N}{w_S}$.

I use equations (9) and (15) to substitute $p(\tilde{s})$, and write the wage ratio as follows:

$$\frac{w_N}{w_S} = \tau + \tau \frac{e^{\lambda_S \tilde{s}}-1}{\lambda_S}(\lambda_S-\lambda_N). \tag{A40}$$

Total differentiating both sides I get:

$$d\frac{w_N}{w_S} = \left(1+\frac{e^{\lambda_S \tilde{s}}-1}{\lambda_S}(\lambda_S-\lambda_N)\right)d\tau + \tau e^{\lambda_S \tilde{s}}(\lambda_S-\lambda_N).\frac{\partial \tilde{s}}{\partial \tau}d\tau \tag{A41}$$

Substituting equation (A33) into equation (A41) for $\frac{\partial \tilde{s}}{\partial \tau}$, and collecting terms, I get:

$$d\frac{w_N}{w_S} = \left(\frac{e^{\lambda_S \tilde{s}}-1}{\lambda_S}(\lambda_S-\lambda_N)\eta+1\right)d\tau, \tag{A42}$$

where $\eta \equiv \frac{\lambda_N(e^{\lambda_S \tilde{s}}-1)}{\lambda_N(e^{\lambda_S \tilde{s}}-1)+\lambda_S(e^{\lambda_N(1-\tilde{s})}-1)e^{\lambda_S \tilde{s}}}>0$, and thus $\left(\frac{e^{\lambda_S \tilde{s}}-1}{\lambda_S}(\lambda_S-\lambda_N)\eta+1\right)>0$, which implies $\frac{\partial \frac{w_N}{w_S}}{\partial \tau}>0$.

Proof of proposition 4.

I decompose the proof of proposition 4 into four steps.

Step 1. The specialization pattern.

Proposition 4 [Specialization] says that, if there is production sharing between one North and multiple identical South, the production structure must be that all South countries provide intermediates to North (no trade between the South countries). In addition, all South nations inside of the production network have the same cutoff stage $\tilde{s}$.

Since all South countries are identical, it is sufficient to consider a one-North, two-South setting. From corollary 2, there will be no production-sharing between the two South. Therefore, if they have any international production-sharing, it must be with North. This means that for any final good, its value-added process only involves two countries: North and one South. Therefore, proposition 1 and the mid-step results of its proof directly apply.

By proposition 1, since South nations have no modern production to start with, there must exist a cutoff stage for each South, such that North produces more downstream and they produce more upstream goods. Denote the two South as S_1, S_2 and their corresponding cutoff stages with North as $\tilde{s}_{S_1}, \tilde{s}_{S_2}$.

Next, I prove $s_1 = s_2$ by contradiction. Suppose $\tilde{s}_{S_1} < \tilde{s}_{S_2}$. The fact that S_2 supplies intermediates to North at stage $\tilde{s}_{S_2}$ implies $Q_N(s) = 0$ for $s \leq s_2$ (proof of proposition 1, step 4 in Appendix). However, since S_1 supplies intermediates to North at stage $\tilde{s}_{S_1}$ and there is no production sharing between the two South nations, $Q_N(s)$ must be positive for $s \in (\tilde{s}_{S_1}, \tilde{s}_{S_2}]$, which contradicts $Q_N(s) = 0$ for $s \leq \tilde{s}_{S_2}$. Therefore, if international production-sharing exists, it must be that there exists a unique stage $\tilde{s} \in (0, 1)$ such that $Q_S(s) > 0$ if and only if $s \in (0, \tilde{s}]$ for joined South countries, and $Q_N(s) > 0$ if and only if $s \in (\tilde{s}, 1]$. Notice this result holds regardless of country size or whether the joined South are completely specialized.

Step 2. Joining.

The first-joined South will join the value chain by producing the most upstream goods for North directly. This will happen when $\theta\tau_{NS_i}$ falls below $\frac{a\lambda_N}{e^{\lambda_N}-1}$. Therefore, as trade costs fall, South that has the lowest trade cost with North will join first, which implies us that $S_j^I = \text{argmin}_{S_i \in C}\{\tau_{NS_i}\}$ for $j = 1$.

For South countries that join later, the results of step 2 of the proof of proposition 1 imply that they cannot join at a middle stage of another nation's production; similarly, due to the triangular inequality, it is straightforward to show that the later-joined South countries cannot join via other countries' cutoff stages either. Therefore, they must join the value chain by producing the most upstream goods for South countries who are already inside the production networks. Suppose that there are already $j-1$ South countries $\{S_1^I, \ldots, S_{j-1}^I\}$ that share production with North; then, country S_i will join the production network when:

$$\theta \leq max_{k \in \{1,\ldots,j-1\}}\{\frac{aw_{S_k^I}}{\tau_{S_k^I S_i}}\}. \tag{A43}$$

Since all joined South share the same cutoff stage $\tilde{s}$ and productivity λ_S, combining equations (15) and (16), I get:

$$\tau_{NS_k^I} w_{S_k^I} = \tau_{NS_{k'}^I} w_{S_{k'}^I}; \tag{A44}$$

therefore (A43) is equivalent to:

$$\theta \leq max_{k\in\{1,\ldots,j-1\}}\{\frac{a\omega}{\tau_{S_k^I S_i}\tau_{NS_{k'}^I}}\}, \tag{A45}$$

where ω is a constant. Denote θ as $\tilde{\theta}_i$ when equality of the above equation holds. When θ falls below $\tilde{\theta}_i$, South i joins the network. The jth successfully joined South must have the highest $\tilde{\theta}_i$ among all outside South countries. Therefore, $S_j^I = \arg\max_{S_i\in C\setminus\{N,S_1^I,\ldots,S_{j-1}^I\}}\{\max_{k\in\{1,\ldots,j-1\}} \frac{a\omega}{\tau_{NS_k^I}\tau_{S_k^I S_i}}\}$ for $j \geq 2$. Rearranging terms, this is equivalent to $S_j^I = \text{argmin}_{S_i\in C\setminus\{N,S_1^I,\ldots,S_{j-1}^I\}}\{\min_{k\in\{1,\ldots,j-1\}} \tau_{NS_k^I}\tau_{S_k^I S_i}\}$ for $j \geq 2$.

Step 3. Rewriting the multi-country problem as a two-country problem.

Equations (9), (15), and (16) can be rewritten as:

$$w_N = \tau w_S + \tau w_S \frac{e^{\lambda_S \tilde{s}} - 1}{\lambda_S}(\lambda_S - \lambda_N), \tag{A46}$$

$$e^{\lambda_N(1-\tilde{s})}\frac{(e^{\lambda_S\tilde{s}} - 1)\tau w_S}{\lambda_S} + (e^{\lambda_N(1-\tilde{s})} - 1)\frac{w_N}{\lambda_N} \equiv 1, \tag{A47}$$

and equations (14) and (13) can be combined as:

$$1 - \tilde{s} = -\frac{1}{\lambda_N}\ln(1 - \frac{\lambda_N L_N}{e^{-\lambda_S\tilde{s}}Q_0/\tau}). \tag{A48}$$

In the multi-South case, using equation (A44), the above three equations can be rewritten as:

$$w_N = \theta\tilde{w}_S + \theta\tilde{w}_S \frac{e^{\lambda_S \tilde{s}} - 1}{\lambda_S}(\lambda_S - \lambda_N), \tag{A49}$$

$$e^{\lambda_N(1-\tilde{s})}\frac{(e^{\lambda_S\tilde{s}} - 1)\theta\tilde{w}_S}{\lambda_S} + (e^{\lambda_N(1-\tilde{s})} - 1)\frac{w_N}{\lambda_N} \equiv 1, \tag{A50}$$

and

$$1 - \tilde{s} = -\frac{1}{\lambda_N}\ln(1 - \frac{\lambda_N L_N}{e^{-\lambda_S\tilde{s}}\tilde{Q}_0/\theta}), \tag{A51}$$

where $\tilde{w}_S = \tau_{NS_k^I} w_{S_k^I}$ for any S_k^I, and $\tilde{Q}_0 = \sum_{S_k^I}(Q_{0,S_k^I}/\tau_{NS_k^I})$.

When all inside South countries are fully specialized on modern production, equation (18) can be rewritten as

$$\sum_{S_k^I}\frac{L_{S_k^I}}{\tau_{NS_k^I}} = \frac{\tilde{Q}_0(1 - e^{-\lambda_S\tilde{s}})}{\lambda_S}. \tag{A52}$$

Suppose that there are countries not fully specialized. By equation (A44), this must be the South country with the highest trade costs with North. I denote this country by S_0^I. In this case, equations (12) and (17) can be rewritten as:

$$\tilde{w_S} = \frac{\tau_{NS_0^I}}{a}. \tag{A53}$$

By (A44), wages of other inside South countries are higher than $\frac{1}{a}$. In other words, only S_0^I is not fully specialized. Therefore:

$$L_S^T = L_{S_0^I}^T = L_{S_0^I} - \frac{\tau_{NS_0^I}(1-e^{-\lambda_S \tilde{s}})}{\lambda_S}\left(\tilde{Q}_0 - \sum_{S_k^I \neq S_0^I}\left(\frac{L_{S_k^I}\lambda_{S_k^I}}{\tau_{NS_k^I}(1-e^{-\lambda_s \tilde{s}})}\right)\right), \tag{A54}$$

with the regularity condition $L_S^T > 0$. Therefore, one can see that the equilibrium can be characterized exactly as the two country case, with τ, w_S, Q_0 and L_S being replaced by θ, $\tilde{w_S}$, $\tilde{Q}_0$ and $\sum_{S_k^I} \frac{L_{S_k^I}}{\tau_{NS_k^I}}$. In other words, all South nations can be viewed as an aggregated South, whose labor supply equals a trade-costs-adjusted sum of labor supply in all countries.

Step 4. Consequences of joining and decreases in global trade costs.

When a new South nation joins, it must be that there is a price gap between inside South countries and the newly-joined South. This implies all inside South countries must be fully specialized before the new one joins. In step 3, I showed that the equilibrium could be characterized as in the two-country model. Therefore, the comparative static results of joining are the same as an increase the labor supply of an aggregated South. Combining equations (13), (14) and (18) to write $\tilde{s}$ as a function of L_S, it is easy to verify $\tilde{s}$ increases when L_S increases. Substituting equation (A46) into equation (A47) to eliminate w_N, it is easy to verify that w_S decreases as $\tilde{s}$ increases, which gives the last part of proposition 4 [Joining].

Conditional on no new entry, the comparative static results, as well as the intuition behind those results of the two-country model directly apply. Therefore, I get proposition 4 [Decreases in trade costs].

Proof of proposition 5.

I first make a slight change in notation to facilitate the proof. Order countries inside the value chain so that λ_c is strictly decreasing in c and index them as $c \in C \equiv \{1, \ldots, \bar{c}\}$. Let $\tilde{s}_{c-1}$ denote the cutoff stage between country c and $c-1$, and τ_{c-1} denote the trade frictions between country c and $c-1$. $M_c \equiv \tilde{s}_c - \tilde{s}_{c-1}$ denotes the measure of stages performed by country c, and $P_c \equiv p(\tilde{s}_c)$ and Q_c denote the price and quantity of the intermediates country c exports, respectively. Then, the equilibrium price, wage and quantity can be characterized by the following equation system:

$$w_c = \theta\tau_{c-1}w_{c-1} + \theta\tau_{c-1}P_{c-1}(\lambda_{c-1} - \lambda_c), \tag{A55}$$

$$P_c = \theta\tau_{c-1}e^{\lambda_c M_c}P_{c-1} + (e^{\lambda_c M_c} - 1)\frac{w_c}{\lambda_c}, \tag{A56}$$

$$M_c \equiv \tilde{s}_c - \tilde{s}_{c-1} = \frac{-1}{\lambda_C} \ln(1 - \frac{\lambda_c L_c}{Q_{c-1}/(\theta \tau_{c-1})}), \tag{A57}$$

$$Q_c = \frac{1}{\theta \tau_{c-1}} e^{-\lambda_c M_c} Q_{c-1}. \tag{A58}$$

The above equations hold for all $c \in C$ with boundary conditions $\tilde{s}_0 = 0$, $\tilde{s}_{\bar{c}} = 1$, $P_0 = 0$, and $P_C = 1$. Next, I show that there exists a country c_1 such that a decrease in θ increases M_c for $c < c_1$ and decreases M_c for $c \geq c_1$. Then, I show that a decrease in θ leads all countries to move up the value chain.

Step 1. If $\theta' < \theta$, there exists $1 \leq c_1 < \bar{c}$ such that $M'_c > M_c$ if $c < c_1$ and $M'_c \leq M_c$ if $c \geq c_1$.

Combining equations (A57) and (A58), I get:

$$M_c = -\frac{1}{\lambda_c} \ln \left(1 - \frac{\lambda_c L_c \theta \tau_{c-1}}{\lambda_{c-1} L_{c-1}} (e^{\lambda_{c-1} M_{c-1}} - 1) \right). \tag{A59}$$

After some algebra one can show that $\frac{\partial M_c}{\partial M_{c-1}} > 0$ and $\frac{\partial M_c}{\partial \theta} > 0$. Since $\theta' < \theta$, equation (A59) implies that if $N'_{c-1} \leq N_{c-1}$ for $c > 1$, then $N'_c < N_c$. Thus, if $N'_1 \leq N_1$, then $N'_c < N_c$ for all $c > 1$, which contradicts the fact that $\sum M_c = 1$. Therefore there must exist $1 < c_1 \leq \bar{c}$ such that $M'_c > M_c$ if $c < c_1$ and $M'_c \leq M_c$ if $c \geq c_1$.

Step 2. If $\theta' < \theta$, then $\tilde{s}'_c > \tilde{s}_c$ for $c \in 1, \ldots, \bar{c} - 1$.

By step 1 of the proof and the definition of N_c, we know that $\tilde{s}'_c > \tilde{s}_c$ if $c < c_1$. If $c_1 = \bar{c}$, the proof is complete. If $c_1 < \bar{c}$, the result of step 1 implies that $c_1 \leq c_2 < \bar{c}$. I denote the first country that has $\tilde{s}'_c < \tilde{s}_c$ by c_2. Since $\sum M'_c = \tilde{s}'_{c_2} + \sum_{c_2+1}^{\bar{c}} M'_c = \tilde{s}_{c_2} + \sum_{c_2+1}^{\bar{c}} M_c = 1$, $\tilde{s}'_{c_2} < \tilde{s}_c$ implies $\sum_{c_2+1}^{\bar{c}} M'_c > \sum_{c_2+1}^{\bar{c}} M_c$, which contradicts $M'_c \leq M_c$ if $c \geq c_1$.