L2. Consumer Choice

Yuan Zi EI037: Microeconomics, Fall 2021

Literature

- MWG (1995), Chapter 2
- Kreps (1990), Chapter 2

2.1 Introduction

Introduction

- We have seen in Lecture 1 that the choice-based approach is weaker and more general than the preference-based approach.
- Therefore, we will first apply the choice-based approach to consumption theory and derive the implications of the weak axiom of revealed preference for the consumer's demand function.
- Then we consider the stronger (but somewhat less general) preference-based approach.

Basic Concepts

- **L commodities**, indexed by $l \in \{1, ..., L\}$
- **consumption plan** (commodity vector): $x = (x_1, ..., x_L) \in \mathbb{R}^L_+ (x_l \geq 0)$
- **Walrasian** budget set: $B = \{x \mid p \cdot x \leq w\}$
- The budget set is convex, i.e., if $x, x' \in B$, then $\alpha x + (1 - \alpha)x' \in B$. Proof?
- The price vector p is orthogonal to the budget hyperplane. Proof?

2.2 Demand Functions and Comparative Statics

Demand Function

If we follow the choice-based approach, then the primitive of our model is the agent's choice rule:

- **demand correspondence** $x(p, w)$
- If $x(p, w)$ is single-valued, we call it the **demand function**.

Demand Function

A Note

1. If we assume that we observe the consumer's demand correspondence, we implicitly assume that we observe his choice behavior for all possible competitive budget sets.
2. If this correspondence is multi-valued, we implicitly assume that the consumer faces the same decision problem (characterized by p and w several times), so that we get to observe all elements of $x(p, w)$.
3. Even if we observe the consumer's choice behavior for all possible competitive budget sets, we do not observe it for all budget sets, in particular not for all budget sets with up to three elements. Hence, the choice-based and the preference-based approaches to consumer theory are not necessarily equivalent.

Demand Function

Assumptions

We impose the following assumptions on $x(p, w)$:

Assumption 2.1 The demand *correspondence* is homogeneous of degree 0. i.e., for all p, w, and $\alpha > 0$

$$x(\alpha p, \alpha w) = x(p, w)$$

- If this assumption was not satisfied, then something would be deeply wrong with the choice-based approach. Why?

Assumption 2.2 The demand *correspondence* $x(p, w)$ satisfies **Walras's law**, i.e., for every p and w we have

$$p \cdot x = w$$

for all $x \in x(p, w)$

- Walras's law requires that the consumer fully expends his resources (over his lifetime).

Assumption 2.3 The consumer's choice rule is single-valued, i.e., $x(p, w)$ is a demand function.

- This assumption is mainly for simplicity.

Comparative Statics

To explore the properties of a given demand function, we analyze how demand changes if prices, p, or wealth, w, change.

Comparative Statics

Wealth effects

- normal goods
- inferior goods
- wealth expansion path
- Engel curve

$\rightarrow$ Illustrate graphically.

Comparative Statics

Price effects

- ordinary goods
- Giffen goods
- offer curve
- demand curve

$\rightarrow$ Illustrate graphically.

2.3 The Weak Axiom and the Compensated Law of Demand

The Weak Axiom of Revealed Preference

Definition 2.1 The demand function $x(p, w)$ satisfies the **weak axiom of revealed preference (WA)**, if the following property holds for any two price-wealth situations (p,w) and (p',w'):

$$p \cdot x(p', w') \leq w \text{ and } x(p', w') \neq x(p, w) \Rightarrow p' \cdot x(p, w) > w'$$

Intuition:

- Suppose that $p \cdot x(p', w') \leq w$ and $x(p', w') \neq x(p, w)$.
- The consumer could have chosen $x(p', w')$ in the situation (p, w), but he chose $x(p, w)$, which does not equal to $x(p', w')$.
- Thus, the consumer has revealed that he prefers $x(p, w)$ to $x(p', w')$.
- Hence, if the consumer chooses $x(p', w')$ in a situation (p', w'), it must be the case that $x(p, w)$ is not affordable in this situation, because otherwise the consumer would have chosen $x(p, w)$.

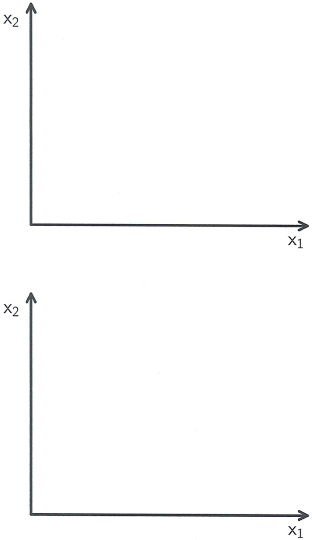

Figure: Weak Axion of Revealed Preference

The Compensated Law of Demand

Slutsky compensation

A price change affects the consumer in two ways:

- relative price changes
- the purchasing power of the consumer's wealth changes

The compensated law of demand focuses on the first effect. In order to isolate this effect, we compensate the consumer's wealth in such a way, that he can still afford the consumption bundle $x(p, w)$ after the price change to p', i.e. his wealth is adjusted to $w' = p' \cdot x(p, w)$. Thus, $\Delta w = \Delta p \cdot x(p, w)$, where $\Delta p = (p' - p)$.

- This is called "Slutsky compensation".

- Illustrate the Slutsky compensation graphically.

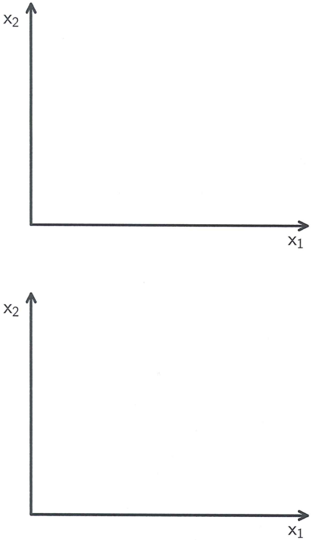

Figure: The Slutsky compensation

The Compensated Law of Demand

Definition 2.2 The **compensated law of demand (CLD)** holds if for any compensated price change from an initial situation (p, w) to a new price-wealth pair $(p', w') = (p', p' \cdot x(p, w))$ we have

$$(p' - p) \cdot [x(p', x') - x(p, w)] \leq 0$$

with strict inequality whenever $x(p, w) \neq x(p', w')$.

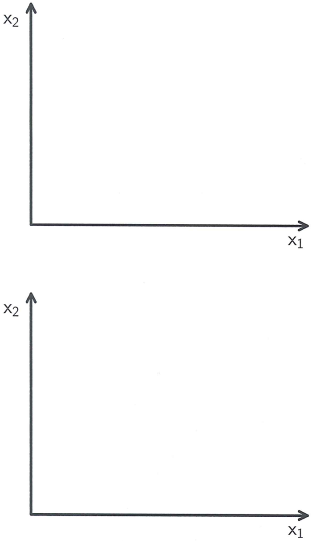

Figure: The Compensated Law of Demand

Proposition 2.1 Suppose that $x(p, w)$ is homogeneous of degree 0 and satisfies Walras' law. Then $x(p, w)$ satisfies the weak axiom if and only if $x(p, w)$ satisfies the compensated law of demand.

Proof: (if) CLD $\rightarrow$ WA

Proposition 2.1 Suppose that $x(p, w)$ is homogeneous of degree 0 and satisfies Walras' law. Then $x(p, w)$ satisfies the weak axiom if and only if $x(p, w)$ satisfies the compensated law of demand.

Proof: (only if) WA $\rightarrow$ CLD

Note that the compensated law of demand is not implied by WA. Show this graphically!

The weak axiom has a further important implication if the demand function is differentiable.

Totally differentiating $x(p, w)$ yields

$$dx = D_p x(p, w) dp^T + D_w x(p, w) dw$$

$\Big[$Matrix notation! x is a column vector $(L \times 1)$, p is a row vector $(1xL)$, $D_p x(p, w)$ is an $L \times L$ matrix, $D_w x(p, w)$ is column vector $(L \times 1)$.$\Big]$

If we consider a compensated price change, then $dw = dp \cdot x(p, w)$. Hence

$$\begin{aligned} dx &= D_p x(p, w) dp^T + D_w x(p, w)[dp \cdot x(p, w)] \\ &= D_p x(p, w) dp^T + D_w x(p, w) x(p, w)^T dp^T \\ &= \left[D_p x(p, w) + D_w x(p, w) x(p, w)^T \right] \cdot dp^T \end{aligned}$$

Substituting this in the compensated law of demand, we get that for any price change dp

$$dp \cdot dx = dp \cdot \left[D_p x(p, w) + D_w x(p, w) x(p, w)^T \right] \cdot dp^T \leq 0$$

The expression in square brackets is an $L \times L$ matrix, which we denote by $S(p, w)$. The (lk)th entry of this matrix is given by

$$s_{lk}(p, w) = \frac{\partial x_l(p, w)}{\partial p_k} + \frac{\partial x_l(p, w)}{\partial w} x_k(p, w).$$

The matrix $S(p, w)$ is called the "Slutsky" or "substitution" matrix, and its elements $s_{lk}(p, w)$ are known as "substitution effects".

Interpretation:

- Consider the effect of a marginal change of price p_k on the demand for good l. If the consumer's wealth is not compensated, this is simply given by

$$\frac{\partial x_l(p, w)}{\partial p_k} dp_k$$

- In order to compensate the consumer for the change in his purchasing power, his wealth changes by the amount $x_k(p, w)dp_k$.
- The effect of this wealth change on the demand for good l is then

$$\frac{\partial x_l(p, w)}{\partial w} x_k(p, w)dp_k$$

- Hence, the total effect of a compensated marginal price change of p_k on good l is given by

$$\left(\frac{\partial x_l(p, w)}{\partial p_k} + \frac{\partial x_l(p, w)}{\partial w} x_k(p, w) \right) dp_k = s_{lk}(p, w)dp_k$$

This derivation is summarized in the following proposition (next slides).

Proposition 2.2 If a differentiable demand function $x(p, w)$ satisfies Walras' law, homogeneity of degree zero, and the weak axiom of revealed preference, then at any point (p, w) the Slutsky matrix $S(p, w)$ satisfies $v \cdot S(p, w) \cdot v^T \leq 0$ for any row vector $v \in \mathbb{R}^L$.

Remarks of Proposition 2.2:

1. A matrix satisfying $v \cdot S(p,w) \cdot v^T \leq 0$ for any $v \in \mathbb{R}^L$ is called **negative semi-definite** and has some interesting properties. In particular, all diagonal elements of this matrix must be non-positive, i.e. $s_{ll}(p,w) \leq 0$. (To see this consider $v = (0,\dots,0,1,0,\dots,0)$.) Hence, **the own substitution effect is always (weakly) negative.**
2. A good is a Giffen good at (p,w) only if it is inferior. To see this note that

$$s_{ll}(p,w) = \frac{\partial x_l(p,w)}{\partial p_l} + \frac{\partial x_l(p,w)}{\partial w} x_l(p,w) \leq 0$$

   A Giffen good has $\frac{\partial x_l(p,w)}{\partial p_l} > 0$. Hence, $\frac{\partial x_l(p,w)}{\partial w} < 0$.
3. The weak axiom does not imply, in general, that the Slutsky matrix is symmetric. In consumer theory this property is the only additional property that comes out of the preference-based approach but which is not implicit in the choice-based approach.