# Lecture 10: Gravity Equation 

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## Introduction

- "A gravity model is typically a log-linear relationship expressing bilateral trade between a pair of countries as a function of the two countries' income level, populations, and distance" Leamer \& Levinsohn (1995).
- The good news: Gravity rules!
- "[These estimates] have produced some of the clearest and most robust empirical finding in economics" Leamer \& Levinsohn (1995)


## Introduction

- Traditional specification (Tinbergen, 1962)

$$
\begin{equation*}
\ln X_{i j}=\beta_{0}+\beta_{1} \ln G D P_{i}+\beta_{2} \ln G D P_{j}+\beta_{3} \ln d i s t_{i j}+\varepsilon_{i j} \tag{1}
\end{equation*}
$$

- Typical estimates:
$-\beta_{1}, \beta_{2}>0, \beta_{3}<0$
$-\beta_{1}, \beta_{2} \simeq 1, \beta_{3} \simeq-1$
- $R^{2}$ around 80-90 \%


## Introduction

France's exports in 2000


## Introduction

France's imports in 2000


## Introduction

Trade within the USA in 1997


## Introduction

## Gravity and the Margins of Trade

|  | $\ln \left(\right.$ Value $\left._{\text {c }}\right)$ | ' | $\ln \left(\right.$ Firms $\left._{\text {c }}\right)$ | $\ln$ Products $_{\text {c }}$ ) | $\ln \left(\right.$ Density $_{\text {c }}$ ) | In(Intensive ${ }_{\text {c }}$ ) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\overline{\text { In( } \text { Distance }^{\text {c }} \text { ) }}$ | -1.37 | I | -1.17 | -1.10 | 0.84 | 0.05 |
|  | 0.17 | , | 0.15 | 0.15 | 0.13 | 0.10 |
| $\ln \left(\right.$ GDP $\left._{c}\right)$ | 1.01 | ' | 0.71 | 0.55 | -0.48 | 0.23 |
|  | 0.04 | 1 | 0.03 | 0.03 | 0.03 | 0.02 |
| Constant | 7.82 | ' | 0.52 | 3.48 | -2.20 | 6.03 |
|  | 1.83 | I | 1.59 | 1.55 | 1.37 | 1.07 |
| Observations | 175 | , | 175 | 175 | 175 | 175 |
| $\mathrm{R}^{2}$ | 0.82 | , | 0.76 | 0.68 | 0.66 | 0.37 |

Notes: Table reports results of country-level OLS regressions of U.S. exports or their components on trading-partners' GDP and great-circle distance (in kilometers) from the United States. Standard errors are noted below each coeficient. Data are for 2002.

Bernard, Redding and Schott (2008) "Multi-product Firms and Trade Liberalization"

## Introduction

It always works, but for a long time, no theory-driven estimations. However:

- Recent theoretical and empirical research improved our understanding of the gravity relationship
- We know why it works...most trade models require gravity to work
- Gravity influenced theoretical analysis (NEG)


## Reading

Head. K. and T. Mayer (2013), "Gravity equations: workhorse, toolkit, and cookbook", CEPR DP 9322

See also the associated webpage

## Theoretical foundations: general formulations

Most theories yield a specification of the form

$$
X_{i j}=\frac{1}{Y} \frac{Y_{i}}{\Omega_{i}} \frac{X_{j}}{\Phi_{j}} \phi_{i j}
$$

(1) the exporter's value of production $Y_{i}=\sum_{j} X_{i j}$
(2) The importer's total expenditures $X_{j}=\sum_{i} X_{i j}$
(3) Bilateral accessibility of $j$ to exporter $i$ (i.e. bil. trade costs) $\phi_{i j}$
(4) "Multilateral resistance"terms: $\Omega_{i}=\sum_{l} \frac{\phi_{i} X_{l}}{\Phi_{I}}$ and $\Phi_{j}=\sum_{l} \frac{\phi_{j i} Y_{l}}{\Omega_{l}}$
$\rightarrow$ Most specific trade models yield such a relationship

## Gravity: specific models

## Theoretical foundations: CES Demands

- Denote $i$ the exporting country producing a set of variety indexed $h$, being consumed in country $j$
- CES utility function is

$$
U_{j}=\left[\int_{i=1}^{N} \int_{h=1}^{n_{i}}\left(b_{i j} q_{i j h}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}
$$

$\rightarrow$ Interpret $b_{i j}$ ? $\sigma$ ?

## Theoretical foundations: CES Demands

- The corresponding demand function for a given product from country $i$ in country $j$ is:

$$
q_{i j}=\frac{b_{i j}^{\sigma-1} p_{i j}^{-\sigma}}{\int_{i=1}^{N} \int_{h=1}^{n_{i}} b_{i j}^{\sigma-1} p_{i j h}^{1-\sigma}} Y_{j}
$$

Defining the welfare based price index: $P_{j}=\left[\int_{i=1}^{N} \int_{h=1}^{n_{i}}\left(\frac{p_{j h}}{b_{i j}}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$

## Theoretical foundations: CES Demands

And using the fact that $X_{i j}=q_{i j} p_{i j}$ we now have an equation defining the value of bilateral imports for a given variety:

$$
X_{i j}=\frac{\left(p_{i j} / b_{j j}\right)^{1-\sigma}}{\rho_{j}^{\sigma-1}} Y_{j}
$$

- Specific models?


## National Product Differentiation (Anderson and Van Wincoop, 2003)

- As in Argmington (1968), each country is the unique source of each product
- Utiliy exhibits CES

$$
U_{j}=\left(\sum_{i} q_{i j} \frac{\sigma-1}{\sigma}\right)^{\frac{\sigma}{\sigma-1}}
$$

- Ïceberg" trade costs: $p_{i j}=p_{i} \tau_{i j}$
- "Phiness" of trade: $\phi_{i j}=\tau_{i j}^{1-\sigma}$


## Theoretical foundations: CES \#1: NPD-AvW

- We get

$$
X_{i j}=\frac{\left(p_{i}\right)^{1-\sigma} \phi_{i j}}{P_{j}^{1-\sigma}} Y_{j}
$$

with the price index: $P_{j}=\left[\sum_{k=1}^{N}\left(p_{k} \tau_{k j}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$

## Theoretical foundations: CES \#1: NPD-AvW

Anderson and Van Wincoop show that, in the special case of symmetric bilateral trade costs, the gravity equation can be rewritten:

$$
X_{i j}=\left(\frac{\tau_{i j}}{P_{j} P_{i}}\right)^{1-\sigma} \frac{Y_{i} Y_{j}}{Y_{W}^{W}}
$$

with $P_{j}=\left[\sum_{i=1}^{N} P_{i}^{\sigma-1} \tau_{i j}^{1-\sigma} \theta_{i}\right]^{\frac{1}{1-\sigma}}, \theta_{i}$ denoting the income share of country $i$
$\rightarrow$ "Multilateral Resistance Indexes

## Theoretical foundations: CES \#1: NPD-AvW

"Multilateral Resistance Indexes"

- Anderson and van Wincoop assume that trade costs are symmetric, and that trade is balanced, then use non-linear least squares to estimate the gravity equation
- Interpretation of these multilateral resistance terms?


## Theoretical foundations: CES \#2: D-S-K

## CES \#2: D-S-K (Dixit-Stiglitz-Krugman)

- DSK assumptions yield gravity
- Each country has $n_{i}$ firms supplying one variety each to the world.
- $n_{i}=\frac{L_{i}}{\sigma F}$ (what is $F$ ?)

We get:

$$
X_{i j}=n_{i j} x_{i j}=\frac{\left(p_{i} \tau_{j j}\right)^{1-\sigma}}{P_{j}^{1-\sigma}} \frac{Y_{j} L_{i}}{\sigma F}
$$

## Theoretical foundations: Helpman, Melitz and Rubinstein (2008)

- Uses Melitz (2003): heterogeneous firms, monopolistic competition
- Selection into exporting: zero trade flows
- Assume productivity defined on $\left[\varphi_{L}, \varphi_{H}\right]$
$\rightarrow$ firms export only if $\varphi \geq \varphi_{i j}^{*}$

Theoretical foundations: Helpman, Melitz and Rubinstein (2008)

Assume that the mass of potential entrants is $N_{i}=\alpha Y_{i}$
Bilateral exports (assuming the same $\mathbf{G}(\varphi)$ everywhere):

$$
X_{i j}=\frac{\tau_{i j}^{1-\sigma}}{P_{j}^{1-\sigma}} Y_{j} N_{i} \int_{\varphi_{i j}}^{\varphi_{H}} p_{i}(\varphi)^{1-\sigma} d G(\varphi)
$$

Theoretical foundations: Helpman, Melitz and Rubinstein (2008)

Prices: $p_{i}(\varphi)=\frac{\sigma}{\sigma-1} \frac{w_{i}}{\varphi}$

$$
\begin{gathered}
X_{i j}=\frac{\sigma}{\sigma-1} \frac{w_{i}^{1-\sigma} \tau_{i j}^{1-\sigma}}{P_{j}^{1-\sigma}} Y_{j} \alpha Y_{i} \int_{\varphi_{i j}}^{\varphi_{H}}\left(\frac{1}{\varphi}\right)^{1-\sigma} d G(\varphi) \text { if } \varphi_{i j}>\varphi_{L} \\
X_{i j}=0 \text { if } \varphi_{i j} \leq \varphi_{L}
\end{gathered}
$$

$\rightarrow 2$ issues: (i) omitted variables (ii) selection bias

- HMR assume that $G(\varphi)$ is Pareto distributed with a shape parameter (inverse measure of heterogeneity)


## Estimation and specification issues

## Estimation methods

- The general form of the gravity equation is

$$
X_{i j}=\frac{1}{Y} \frac{Y_{i}}{\Phi_{i}} \frac{Y_{j}}{\Phi_{j}} \phi_{i j}
$$

- Taking logs:

$$
\ln X_{i j}=\ln Y+\ln \frac{Y_{i}}{\Phi_{i}}+\ln \frac{Y_{j}}{\Phi_{j}}+\ln \phi_{i j}
$$

## Estimation methods

$\ln X_{i j}=\ln Y+\ln \frac{Y_{i}}{\Phi_{i}}+\ln \frac{Y_{j}}{\Phi_{j}}+\ln \phi_{i j}$

- Tradition: using In GDPs (and possibly other variables, such as GDP per capita) as proxies for $\ln \frac{Y_{i}}{\Phi_{i}}$ and $\ln \frac{Y_{j}}{\Phi_{j}}$
- With GDPs only, omitted variable bias: "gold medal mistake"
- What bias? Solution?


## Estimation methods

Method matters for the interpretation of coefficients. Take AvW (2003):

$$
X_{i j}=\frac{Y_{i} Y_{j}}{Y}\left(\frac{\phi_{j j}}{\Phi_{i} \phi_{j}}\right)
$$

- Bilateral trade is increasing in the remoteness. ${ }^{\circ}$ the pair $\frac{1}{\Phi i \phi_{j}}$. When omitted, all $\phi_{i j}$ variables that affect trade positively will tend to be biased downwards if they are negatively correlated with remoteness, and vice-versa
- With $\phi_{i j}=\tau_{i j}^{1-\sigma}$, estimation of $\sigma$ is possible if one has data on direct price shifter like tariffs


## Estimation methods

Main solutions:
(1) Include proxies for $\Phi_{i}$ and $\Phi_{j}$ such as "Remoteness indexes", e.g the inverse of Harris market potential $\sum_{i} Y_{i} /$ Dist $_{j i}$
(2) Ratio-type estimation
(3) More fancy approximation of the multilateral resistance terms
(4) Fixed effects estimations

## Estimation methods

Main solutions:
(1) Include proxies for $\Phi_{i}$ and $\Phi_{j}$ such as "Remoteness indexes", e.g the inverse of Harris market potential $\sum_{i} Y_{i} /$ Dist $_{j i}$

Problem: doesn't take the theory seriously... why?

## Estimation methods

## "Ratio-type Gravity"

- One can use the multiplicative structure of the gravity model to get rid of trouble terms.
- Bilateral "relative" imports by country j from country i for a given industry / year (Head and Mayer, 2001)

$$
\begin{equation*}
\frac{X_{i j}}{X_{j j}}=\frac{n_{i}}{n_{j}}\left(\frac{p_{i}}{p_{j}}\right)^{1-\sigma}\left(\frac{\phi_{i j}}{\phi_{j j}}\right) \tag{8}
\end{equation*}
$$

## Estimation methods

## "Ratio-type Gravity"

- Problem: we need to observe "trade with self"
- But these manipulations can be done with any reference country (Martin et al., 2008)

$$
\frac{x_{i j}}{X_{U S j}}=\frac{n_{i}}{n_{U S}}\left(\frac{p_{i}}{p_{U S}}\right)^{1-\sigma}\left(\frac{\phi_{i j}}{\phi_{U S j}}\right)
$$

## Estimation methods

"Bonus Vetus OLS", Baier and Bergstrand, 2009

- B \& B Approximate the multilateral resistance terms using a first-order log linear Taylor series expansion. They show that if trade costs are symmetric:

$$
\begin{aligned}
& \ln \Pi_{i}=\sum_{j=1}^{N} \theta_{j} \ln \tau_{i j}-\frac{1}{2} \sum_{k=1}^{N} \sum_{m=1}^{N} \theta_{k} \theta_{m} \tau_{k m}, i=2, \ldots, N \\
& \ln P_{j}=\sum_{i=1}^{N} \theta_{i} \ln \tau_{i j}-\frac{1}{2} \sum_{k=1}^{N} \sum_{m=1}^{N} \theta_{k} \theta_{m} \tau_{k m}, j=2, \ldots, N
\end{aligned}
$$

## Estimation methods

## Fixed effects estimation

- Include in the estimation Fixed effects of the dimension of $\Phi_{i}$ and $\Phi_{j}$
- In a cross-section, means including importer and exporter FE.
- With panel data, importer $\times$ year and exporter $\times$ year FE
- No structural assumption on the underlying model, and can capture potential country-specific determinants of trade
- Problem: computational difficulties (imagine a model with 150 countries and 50 years...)
- Use reg2hdfe or reg3hdfe


## Estimation methods

## Fixed effects estimation

Issue with these approaches?

- What if we want to identify country-specific effects (income elasticities, effect of financial crises, effect of institutional determinants)
- A possibility is to estimate the specific with FE, and then regress the FE on the (country-specific) variable of interest


## Method matters: Rose (2004), AER

Table 1: Benchmark Results

|  | Default | No Industrial Countries | Post '70 | With Country Effects |
| :---: | :---: | :---: | :---: | :---: |
| Both in GATT/WTO | $\begin{array}{r} \hline-.04 \\ (.05) \\ \hline \end{array}$ | $\begin{gathered} -.21 \\ (.07) \\ \hline \end{gathered}$ | $\begin{aligned} & \hline-.08 \\ & (.07) \\ & \hline \end{aligned}$ | $\begin{gathered} .15 \\ (.05) \end{gathered}$ |
| One in GATT/WTO | $\begin{array}{r} -.06 \\ (.05) \\ \hline \end{array}$ | $\begin{array}{r} -.20 \\ (.06) \\ \hline \end{array}$ | $\begin{array}{r} \hline-.09 \\ (.07) \\ \hline \end{array}$ | $\begin{gathered} .05 \\ (.04) \\ \hline \end{gathered}$ |
| GSP | $\begin{array}{r} .86 \\ (.03) \\ \hline \end{array}$ | $\begin{array}{r} .04 \\ (.10) \\ \hline \end{array}$ | $\begin{array}{r} .84 \\ (.03) \\ \hline \end{array}$ | $\begin{array}{r} .70 \\ (.03) \\ \hline \end{array}$ |
| $\begin{array}{r} \mathrm{Log} \\ \text { Distance } \\ \hline \end{array}$ | $\begin{aligned} & -1.12 \\ & (.02) \\ & \hline \end{aligned}$ | $\begin{array}{r} -1.23 \\ (.03) \\ \hline \end{array}$ | $\begin{array}{r} -1.22 \\ (.02) \\ \hline \end{array}$ | $\begin{array}{r} -1.31 \\ (.02) \\ \hline \end{array}$ |
| Log product Real GDP | $\begin{array}{r} .92 \\ (.01) \\ \hline \end{array}$ | $\begin{array}{r} .96 \\ (.02) \\ \hline \end{array}$ | $\begin{aligned} & .95 \\ & (.01) \\ & \hline \end{aligned}$ | $\begin{array}{r} .16 \\ (.05) \\ \hline \end{array}$ |
| Log product Real GDP p/c | $\begin{array}{r} .32 \\ (.01) \\ \hline \end{array}$ | $\begin{array}{r} .20 \\ (.02) \\ \hline \end{array}$ | $\begin{array}{r} .32 \\ (.02) \\ \hline \end{array}$ | $\begin{gathered} .54 \\ (.05) \\ \hline \end{gathered}$ |
| Regional FTA | $\begin{aligned} & 1.20 \\ & (.11) \end{aligned}$ | $\begin{array}{r} 1.50 \\ (.15) \\ \hline \end{array}$ | $\begin{array}{r} 1.10 \\ (.12) \\ \hline \end{array}$ | $\begin{gathered} .94 \\ (.13) \\ \hline \end{gathered}$ |
| Currency <br> Union | $\begin{array}{r} 1.12 \\ (.12) \\ \hline \end{array}$ | $\begin{array}{r} 1.00 \\ (.15) \\ \hline \end{array}$ | $\begin{array}{r} 1.23 \\ (.15) \\ \hline \end{array}$ | $\begin{array}{r} 1.19 \\ (.12) \\ \hline \end{array}$ |
| Common <br> Language | $\begin{array}{r} .31 \\ (.04) \\ \hline \end{array}$ | $\begin{gathered} .10 \\ (.06) \\ \hline \end{gathered}$ | $\begin{gathered} .35 \\ (.04) \\ \hline \end{gathered}$ | $\begin{gathered} .27 \\ (.04) \\ \hline \end{gathered}$ |
| $\begin{array}{rr} \hline & \begin{array}{r} \text { Land } \\ \text { Border } \end{array} \\ \hline \end{array}$ | $\begin{array}{r} 53 \\ (.11) \\ \hline \end{array}$ | $\begin{array}{r} .72 \\ (.12) \\ \hline \end{array}$ | $\begin{array}{r} .69 \\ (.12) \\ \hline \end{array}$ | $\begin{gathered} .28 \\ (.11) \end{gathered}$ |

## Method matters: Baier and Bergstrand (2009), JIE

## Table 1

Estimation results: Canada-U.S

| Parameters | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ | $(7)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | OLS w/o | A-vW | A-vW | OLS with | Fixed | A-vW | OLS with |
|  | MR terms | NLS-2 | NLS-3 | MR terms | effects | NLS-2-a | MR terms-a |
| $-\rho(\sigma-1)$ for | -1.06 | -0.79 | -0.82 | -0.82 | -1.25 | -0.92 | -1.02 |
| distance | $(0.04)$ | $(0.03)$ | $(0.03)$ | $(0.04)$ | $(0.04)$ | $(0.03)$ | $(0.03)$ |
| $-\alpha(\sigma-1)$ for | -0.71 | -1.65 | -1.59 | -1.11 | -1.54 | -1.65 | -1.24 |
| border | $(0.06)$ | $(0.08)$ | $(0.08)$ | $(0.07)$ | $(0.06)$ | $(0.07)$ | $(0.07)$ |
| Avg. error terms |  |  |  |  |  |  |  |
| US-US | -0.21 | 0.06 | 0.06 | 0.39 | 0.00 | 0.05 | 0.27 |
| CA-CA | 1.95 | -0.17 | -0.02 | -0.34 | 0.00 | -0.22 | -0.23 |
| US-CA | 0.00 | -0.05 | -0.04 | -0.50 | 0.00 | -0.04 | -0.35 |
| $R^{2}$ | 0.42 | n.a. | n.a. | 0.36 | 0.66 | n.a. | 0.60 |
| No. of obs. | 1511 | 1511 | 1511 | 1511 | 1511 | 1511 | 1511 |

Numbers in parentheses are standard errors of the estimates. n.a. denotes not applicable.

## Method matters: Martin, Mayer, Thoenig (2008), REStud

Impact of militarized interstate dispute on trade

|  | Dependent variables |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | In imports |  | $\ln m_{i j t} / m_{\text {iut }}$ |  |
|  | Model (1) | Model (2) | Model (3) | Model (4) |
| ln GDP origin | $\begin{gathered} 0.959^{* * * *} \\ (0.006) \end{gathered}$ | $\begin{gathered} 0.940^{* * *} \\ (0.007) \end{gathered}$ | $\begin{gathered} 1.001^{* * *} * \\ (0.007) \end{gathered}$ | $\begin{gathered} 0.976^{* * *} \\ (0.008) \end{gathered}$ |
| 1 n GDP destination | $\begin{gathered} 0.847^{*} * * \\ (0.006) \end{gathered}$ | $\begin{gathered} 0.846^{* * *} \\ (0.007) \end{gathered}$ | - | - |
| In distance | $\begin{gathered} -1.008 * * * \\ (0.017) \end{gathered}$ | $\begin{gathered} -0.991 * * * \\ (0.019) \end{gathered}$ | $\begin{gathered} -1 \cdot 188^{* * *} \\ (0.018) \end{gathered}$ | $\begin{gathered} -1 \cdot 158^{* * *} \\ (0.019) \end{gathered}$ |
| Contiguity | $\begin{gathered} 0.452^{* * *} \\ (0.075) \end{gathered}$ | $\begin{gathered} 0.412^{* * *} \\ (0.078) \end{gathered}$ | $\begin{gathered} 0.663^{* * *} * \\ (0.066) \end{gathered}$ | $\begin{gathered} 0.680 * * * \\ (0.069) \end{gathered}$ |
| Similarity in language index | $\begin{gathered} 0.331 * * * \\ (0.070) \end{gathered}$ | $\begin{gathered} 0.301^{* * *} \\ (0.074) \end{gathered}$ | $\begin{aligned} & 0.128^{* *} \\ & (0.062) \end{aligned}$ | $\begin{aligned} & 0.112^{*} \\ & (0.065) \end{aligned}$ |
| Colonial link ever | $\begin{gathered} 1 \cdot 121^{* * *} \\ (0.088) \end{gathered}$ | $\begin{gathered} 1.060 * * * \\ (0.093) \end{gathered}$ | $\begin{gathered} 0 \cdot 302 * * * \\ (0.061) \end{gathered}$ | $\begin{gathered} 0.257 * * * \\ (0.063) \end{gathered}$ |
| Common colonizer post-1945 | $\begin{gathered} 0.568^{* * *} \\ (0.058) \end{gathered}$ | $\begin{gathered} 0.499 * * * \\ (0.064) \end{gathered}$ | $\begin{gathered} 0.545 * * * \\ (0.063) \end{gathered}$ | $\begin{gathered} 0.450 * * * \\ (0.069) \end{gathered}$ |
| Preferential trade arrangement | $\begin{gathered} 0.545^{*} * * \\ (0.049) \end{gathered}$ | $\begin{gathered} 0.539^{* * *} \\ (0.052) \end{gathered}$ | $\begin{gathered} 0.441^{* * *} \\ (0.049) \end{gathered}$ | $\begin{gathered} 0.426^{* * *} \\ (0.053) \end{gathered}$ |
| Number of GATT/WTO members | $\begin{gathered} 0 \cdot 204^{* * *} \\ (0.021) \end{gathered}$ | $\begin{gathered} 0.223^{* * *} \\ (0.022) \end{gathered}$ | $\begin{gathered} 0.337 * * * \\ (0.034) \end{gathered}$ | $\begin{gathered} 0.364^{*} * * \\ (0.036) \end{gathered}$ |
| One communist regime among partners | $\begin{gathered} -0.399^{* * *} \\ (0.032) \end{gathered}$ | $\begin{gathered} -0.422 * * * \\ (0.034) \end{gathered}$ | $\begin{gathered} -0.720^{* * *} \\ (0.045) \end{gathered}$ | $\begin{gathered} -0.767^{* * *} \\ (0.045) \end{gathered}$ |
| bil. MID + 0 years | $\begin{gathered} -0.245^{* * *} \\ (0.059) \end{gathered}$ | $\begin{gathered} -0.244^{* * *} \\ (0.044) \end{gathered}$ | $\begin{gathered} -0.485 * * * \\ (0.036) \end{gathered}$ | $\begin{gathered} -0.434^{* * *} \\ (0.032) \end{gathered}$ |
| Equation |  |  | Yuan | (0) 36/56 |

## Estimation methods

## Other problem: heteroscedasticity

Pointed out by Santos Silva and Tenreyro (2006)

- Problems with log-specification: heteroskedasticity
... which may lead to inconsistent OLS estimates of log-linearized models due to heteroscedasticity
- Why? Because the expected value of the log of a random var. depends on its mean and on higher order moments of the distribution
- More precisely, $E[\log (u \mid X)] \simeq \log [E(u \mid X)]-\frac{\operatorname{Var}(u \mid X)}{2 E(u \mid X)}$
- Poisson pseudo-maximum likelihood estimations (PPML) or Gamma PML


## Estimation methods

## Problem with log-specification: zeros

- Log of zero does not exist...but we observe zeros in trade data
- At the aggregated level only $50 \%$ of possible trade lines are filled
- What to do with these zeros?


## Estimation methods: zero trade flows

- What to do with these zeros?
- Drop them? Selection bias
- Assume they are small positive trade flows: replace all observations by $x+1$ : inconsistent estimator
- Use an estimator that allows the inclusion of zeros (PPML or Tobit)
- Control for selection bias? Heckman model : need an exclusion variable (which explains the selection but not the value of traded flow). Problems with Heckman model?
- Related question: where do the zeros come from? "Real"zeros or statistical issue?


## Gravity equations: applications

## The gravity equation: what for?

-Estimate / evaluate the impact of trade barriers:

- Direct estimation: influence of RTAs, tariffs, exchange rate volatility
- Estimate parameters of trade model $(\sigma)$
- Measuring border effects
- Proxies of trade costs: influence of distance, cultural proximity (language, colonial links, migrations, etc...)


## The gravity equation: what for?

- Measuring the influence of distance
- Consider the real extent of globalization: is the world really flat?
- Evaluating (overall) trade barriers
- Track informal barriers (norms, administrative barriers, etc)
$\rightarrow$ Border effects
- Measuring the impact of joining WTO, FTA, Monetary Union
- Estimating the effect of the current financial crisis


## Meta-analysis of gravity variables (Head and Mayer, 2013)

Table 4: Estimates of typical gravity variables

|  | All Gravity |  |  |  | Structural Gravity |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Estimates: | median | mean | s.d. | $\#$ | median | mean | s.d. | $\#$ |
| Origin GDP | .97 | .98 | .42 | 700 | .86 | .74 | .45 | 31 |
| Destination GDP | .85 | .84 | .28 | 671 | .67 | .58 | .41 | 29 |
| Distance | -.89 | -.93 | .4 | 1835 | -1.14 | -1.1 | .41 | 328 |
| Contiguity | .49 | .53 | .57 | 1066 | .52 | .66 | .65 | 266 |
| Common language | .49 | .54 | .44 | 680 | .33 | .39 | .29 | 205 |
| Colonial link | .91 | .92 | .61 | 147 | .84 | .75 | .49 | 60 |
| RTA/FTA | .47 | .59 | .5 | 257 | .28 | .36 | .42 | 108 |
| EU | .23 | .14 | .56 | 329 | .19 | .16 | .5 | 26 |
| CUSA/NAFTA | .39 | .43 | .67 | 94 | .53 | .76 | .64 | 17 |
| Common currency | .87 | .79 | .48 | 104 | .98 | .86 | .39 | 37 |
| Home | 1.93 | 1.96 | 1.28 | 279 | 1.55 | 1.9 | 1.68 | 71 |

Notes: The number of estimates is 2508 , obtained from 159 papers. Structural gravity refers here to some use of country fixed effects or ratio-type method.

## The impact of distance: Disdier \& Head (2008, Restat)

Meta Analysis: examine 1467 distance effects estimated in 103 papers
Finding: the estimated negative impact of distance on trade actually rose aroun the middle of the XXth century

Solid point: highest $\mathbf{R}^{2}$ in the paper


## The gravity equation: what for?

## Impact of currency unions on bilateral trade: Rose (2000, Economic Policy)

- Very simple analysis of the impact of CUs
- Focus on all existing unions (but discussion oriented toward the EMU)
- Very basic methodology: create a dummy CU. plug it into a gravity equation, estimate with OLS

$$
\ln \left(x_{i j}\right)=\alpha_{1} \ln G D P_{i}+\alpha_{2} \ln G D P_{j}+\alpha_{3} \ln D i s t_{i j}++\alpha_{4} C U_{i j}+X_{i j}+\varepsilon_{i j}
$$

Belonging to a CU multiplies bilateral trade by $\exp \left(\alpha_{4}\right)$

## The impact of CU: Rose (2000, Economic Policy)

Belonging to a CU multiplies trade by $\mathrm{e}^{1.21}=3.35$ !
Problems with Rose's methodology?

|  | 1970 | 1975 | 1980 | 1985 | 1998 | Pooled |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Currency Union $\gamma$ | .87 | 1.28 | 1.09 | 1.40 | $(1.51$ | 1.21 |
|  | $(.43)$ | $(.41)$ | $(.26)$ | $(.27)$ | $(27)$ | $(.14)$ |
| Exchange Rate Volatility $\delta$ | -.062 | .001 | -.060 | -.028 | -.009 | 017 |
|  | $(.012)$ | $(.008)$ | $(.010)$ | $(.005)$ | $(.002)$ | $(.002)$ |
| Output $\mathrm{b}_{1}$ | .77 | .81 | .81 | .80 | .83 | .80 |
|  | $(.02)$ | $(.01)$ | $(.01)$ | $(.01)$ | $(.01)$ | $(.01)$ |
| Output/Capita $\mathrm{b}_{2}$ | .65 | .66 | .61 | .66 | .73 | .66 |
|  | $(.03)$ | $(.03)$ | $(.02)$ | $(.02)$ | $(.02)$ | $(.01)$ |
| Distance $\mathrm{b}_{3}$ | -1.09 | -1.15 | -1.03 | -1.05 | -1.12 | -1.09 |
|  | $(.05)$ | $(.04)$ | $(.04)$ | $(.04)$ | $(.04)$ | $(.02)$ |
| Contiguity $\mathrm{b}_{4}$ | .48 | .36 | .73 | .52 | .63 | .53 |
|  | $(.21)$ | $(.19)$ | $(.18)$ | $(.18)$ | $(.18)$ | $(.08)$ |
| Language $\mathrm{b}_{5}$ | .56 | .36 | .28 | .36 | .50 | .40 |
|  | $(.10)$ | $(.10)$ | $(.09)$ | $(.08)$ | $(.08)$ | $(.04)$ |
| FTA $\mathrm{b}_{6}$ | .87 | 1.02 | 1.26 | 1.21 | .67 | .99 |
|  | $(.16)$ | $(.21)$ | $(.16)$ | $(.17)$ | $(.14)$ | $(.08)$ |
| Same Nation $\mathrm{b}_{7}$ | 1.02 | 1.37 | 1.12 | 1.36 | .88 | 1.29 |
|  | $(.74)$ | $(.59)$ | $(.38)$ | $(.64)$ | $(.52)$ | $(.26)$ |

## The gravity equation: the effect of currency unions

## Obvious critics:

- Awkward data: most of the common currency pairs involved nations that were very small / very poor
- Omitted variables: that are pro-trade and correlated with CU dummy; biases the estimates upward (eg trust, peaceful relations, etc)
- Reverse causality: large bilateral flows cause a CU...
- Model Mis-specification



## The impact of CU: Rose and Van Wincoop (2001)

Table 1: Impact of Currency Union on International Trade, 1970-1995

| Currency Unign Dumux | $\begin{aligned} & 1.38 \\ & (.19) \\ & \hline \end{aligned}$ | $\begin{array}{r} .86 \\ (.19) \\ \hline \end{array}$ |
| :---: | :---: | :---: |
| Log Distance | $\begin{gathered} -1.06 \\ (.03) \\ \hline \end{gathered}$ | $\begin{aligned} & -1.31 \\ & (.03) \\ & \hline \end{aligned}$ |
| Log Product Real <br> GDP | $\begin{gathered} .94 \\ (.01) \\ \hline \end{gathered}$ | $\begin{aligned} & 1.06 \\ & (.04) \\ & \hline \end{aligned}$ |
| Common Language <br> Dummy | $\begin{gathered} .56 \\ (.06) \\ \hline \end{gathered}$ | $\begin{gathered} .48 \\ (.06) \\ \hline \end{gathered}$ |
| Common Land Border Dummy | $\begin{gathered} .63 \\ (.12) \\ \hline \end{gathered}$ | $\begin{gathered} .30 \\ (.13) \\ \hline \end{gathered}$ |
| Free Trade Agreement Drummy | $\begin{aligned} & 1.09 \\ & (.10) \\ & \hline \end{aligned}$ | $\begin{array}{r} .46 \\ (.12) \\ \hline \end{array}$ |
| Common Colonizer <br> Dummy | $\begin{gathered} .41 \\ (.08) \\ \hline \end{gathered}$ | $\begin{gathered} .68 \\ (.08) \\ \hline \end{gathered}$ |
| Ex-Colony/ Colonizer Dummy | $\begin{aligned} & 1.97 \\ & (.13) \end{aligned}$ | $\begin{aligned} & 1.74 \\ & (.13) \end{aligned}$ |
| Political Union Dummy | $\begin{gathered} .95 \\ (.37) \\ \hline \end{gathered}$ | $\begin{gathered} .81 \\ (.32) \\ \hline \end{gathered}$ |
| Log Product Real GDP/capita | $\begin{gathered} .48 \\ (.02) \\ \hline \end{gathered}$ |  |
| Number landlocked | $\begin{aligned} & -.32 \\ & (.04) \\ & \hline \end{aligned}$ |  |
| Log of Land Area Product | $\begin{gathered} -.15 \\ (.01) \\ \hline \end{gathered}$ |  |
| RMSE | 1.97 | 1.74 |
| $\mathrm{R}^{2}$ | . 64 | . 72 |
| Observations | 31,101 | 31,101 |
|  | $\begin{aligned} & \text { Time } \\ & \text { Effects } \end{aligned}$ | Time, Country Effects |

«Gold Medal» Mistake: Rose (2000) omitted country FE The effect drops... the estimates were severely biased upwards ...but the effect is still very large

Without FE: Trade * 3.97
With FE: Trade*2.36

Other ways to control for omitted variable bias?

## The gravity equation: the effect of currency unions

## Omitted variables

- Simple solution: dyadic fixed effects
- Will control for anything that does not vary over time and affects bilateral trade.

Glick and Rose (2000, European Economic Review): coefficient decrease to 0.65: CU increase trade by 90 \%

## The gravity equation: the effect of currency unions

## Omitted variable

- Volker Nitsch ("Honey, I shrunk the currency union effect on trade")
- Finds that exit have a very negative impact, entry have an insignificant one
- Exit often go together with time-varying troubles (political, etc)


## The gravity equation: the effect of currency unions

## Self-selection

- CU pairs are very unusual countries: very small country, nearby a large one, that trade a lot
- the "experiment" CU is by no way random (self selection)
- Solution: Matching: find for each pair of country the most proximate country pair which is not a CU


## Self-selection

- Persson (2001)

First Step: Probability of
joining a CU

Table 2. Propensity score (logit parameter estimates)

| Output | -0.240 |
| :--- | ---: |
|  | $(0.033)$ |
| Output/per capita | -0.168 |
|  | $(0.058)$ |
| Distance | -1.016 |
|  | $(0.088)$ |
| Continguity | -0.390 |
| Language | $(0.278)$ |
|  | 1.743 |
| Free trade area | $(0.208)$ |
|  | -1.431 |
| Same nation | $(0.292)$ |
|  | 6.246 |
| Same colonizer | $(0.546)$ |
| Colonial relation | 1.401 |
|  | $(0.203)$ |
| No. Obs. | -1.817 |
| Pseudo R | $(0.695)$ |

Note: Standard errors in brackets.

## The gravity equation: the effect of RTAs

## Effects of Regional Trade Agreements

- Previous discussion on CU also applied to RTAs

Table 4
Panel gravity equations in levels using various specifications

| Variable | (1) No fixed or time <br> effects | (2) With time <br> effects | (3) With bilateral fixed <br> effects | (4) With time and bilateral <br> fixed effects |
| :--- | :--- | :--- | :--- | :--- |
| $\ln \mathrm{RGDP}_{i}$ | $0.95(217.50)$ | $0.97(230.98)$ | $0.71(34.54)$ | $1.27(47.16)$ |
| $\ln \mathrm{RGDP}_{j}$ | $0.94(224.99)$ | $0.97(235.43)$ | $0.58(26.57)$ | $1.22(41.60)$ |
| ${\ln \mathrm{DIST}_{i j}}^{\mathrm{ADJ}_{i j}}$ | $-1.03(-79.09)$ | $-1.01(-78.60)$ |  |  |
| LANG $_{i j}$ | $0.41(8.23)$ | $0.38(7.28)$ |  |  |
| FTA $_{i j}$ | $0.63(19.06)$ | $0.58(17.73)$ |  | $0.68(14.27)$ |
| RMSE $^{0.13(3.73)}$ | 1.9270 | $0.27(7.19)$ | $0.51(10.74)$ |  |
| Overall $R^{2}$ | 0.6575 | 1.8601 |  | 0.2268 |
| Within $R^{2}$ |  | 0.6809 |  | 47,081 |
| No. observations | 47,081 | 47,081 | 47,081 |  |

$t$-statistics are in parentheses. The dependent variable is the (natural $\log$ of the) real bilateral trade flow from $i$ to $j$. Coefficient estimates for various fixed/time effects are not reported for brevity.

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