Factor Proportion Theory

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Today's lecture

- Ricardo-Viner model
  - Basic environment
  - Comparative statics
- Two-by-Two Heckscher-Ohlin model
  - Basic environment
  - Classic results
    - Factor Price Equalization Theorem
    - Stolper-Samuelson (1941) Theorem
    - Rybczynski (1965) Theorem
- Heckscher-Ohlin Theorem
- Heckscher-Ohlin-Vanek Theorem

Factor Proportion Theory

- The law of comparative advantage establishes the relationship between relative autarky prices and trade flows
  - But where do relative autarky prices come from?
- Factor proportion theory emphasizes **factor endowment differences** as the source of comparative advantage
- **Key elements**:
  1. Countries differ in terms of factor abundance [i.e *relative* factor supply]
  2. Goods differ in terms of factor intensity [i.e *relative* factor demand]
- Interaction between 1 and 2 will determine differences in relative autarky prices, and in turn, specialization and the pattern of trade

Factor Proportion Theory

- In order to shed light on factor endowments as a source of CA, we assume that:
  1. Production functions are identical around the world
  2. Households have identical homothetic preferences around the world
- We focus on two special models:
  - **Ricardo-Viner** with 2 goods, 1 "mobile" factor (labor) and 2 "immobile" factors (sector-specfic capital)
  - **Heckscher-Ohlin** with 2 goods and 2 "mobile" factors (labor and capital)
- The second model is often thought of as a long-run version of the first (Neary 1978)
  - In the case of Heckscher-Ohlin, we need to consider, what is the time horizon such that one can think of total capital as fixed in each country, though freely mobile across sectors?
- Then we study a many-goods, many-country extension:
  - **Heckscher-Ohlin-Vanek**

Ricardo-Viner Model

Basic environment

- Consider an economy with:
  - Two goods, $g = 1, 2$
  - Three factors with endowments l, k_1, k_2
- Output of good g is given by

$$y_g = f^g(l_g, k_g)$$

where:

  - l_g is the endogenous amount of labor in sector g
  - f^g is homogeneous of degree 1 in (l_g, k_g)
- **Comments:**
  - l is a "mobile" factor in the sense that it can be employed in all sectors
  - k_1 and k_2 are "immobile" factors in the sense that they can only be employed in one of the sectors
  - The model is isomorphic to DRS model: $y_g = f^g(l_g)$ with $f_l^g > 0, f_{ll}^g < 0$
  - Payments to specific factors under CRS$\equiv$profits under DRS

Ricardo-Viner Model

Equilibrium (I): small open economy

- We denote by:
  - p_1, p_2 the prices of goods 1 and 2
  - $w, r_1.$ and r_2 the prices of l, k_1 and k_2
- For now, (p_1, p_2) is exogenously given: **"small open economy"**
  - So no need to look at good market clearing
- Profit maximization:

$$p_g f_l^g(l_g, k_g) = w \tag{1}$$

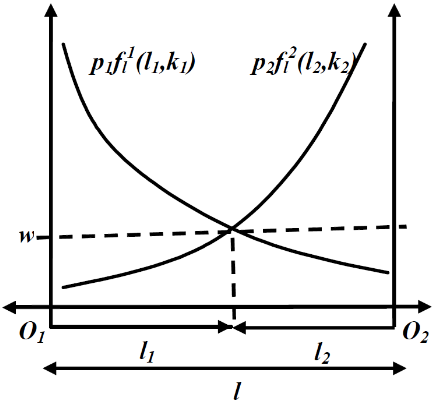
$$p_g f_k^g(l_g, k_g) = r_g \tag{2}$$

- Labor market clearing:

$$l = l_1 + l_2 \tag{3}$$

Ricardo-Viner Model

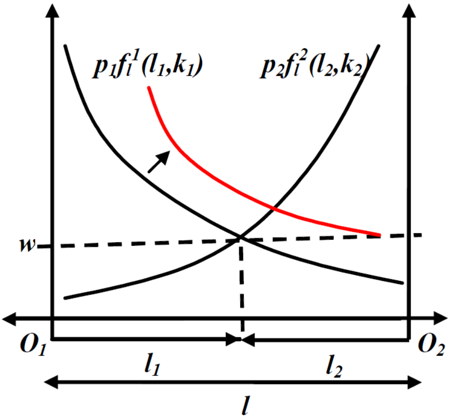
Graphical analysis



- Equations (1) and (3) jointly determine labor allocation and wage
- How do we recover payments to the specific factor from this graph?

Ricardo-Viner Model

Comparative statics



- Consider a TOT shock such that p_1 increases:
  - $w \nearrow$, $l_1 \nearrow$ and $l_2 \searrow$
  - Condition (2) $\implies$ $r_1/p_1 \nearrow$ whereas r_2 (and r_2/p_1) $\searrow$

Ricardo-Viner Model

Comparative statics

- One can use the same type of arguments to analyze consequences of:
  - Productivity shocks
  - Changes in factor endowments
- In all cases, results are intuitive:
  - Dutch disease (boom in export sectors, bids up wages, which leads to a contraction in the other sectors)
- Easy to extend the analysis to more than 2 sectors
  - Plot labor demand in one sector vs. rest of the economy

Ricardo-Viner Model

Equilibrium (II): two-country world

- Predictions on the pattern of trade in a two-country world depend on whether differences in factor endowments come from:
  - Differences in the relative supply of specific factors
  - Differences in the relative supply of mobile factors
- Accordingly, any change in factor prices is possible as we move from autarky to free trade

Two-by-Two Heckscher-Ohlin Model

Basic environment

- Consider an economy with:
  - Two goods, $g = 1, 2$
  - Two factors with endowments l and k
- Output of good g is given by

$$y_g = f^g(l_g, k_g)$$

  where

  - l_g, k_g are the (endogenous) amounts of labor and capital in sector g
  - f^g is homogeneous of degree 1 in (l_g, k_g)

Two-by-Two Heckscher-Ohlin Model

The dual approach

- $c_g(w, r)$ is unit cost function in sector g

$$c_g(w, r) = \min_{l,k} \{wl + rk \,|\, f^g(l, k) \geqslant 1\}$$

  where w and r is the price of labor and capital respectively
- $a_{fg}(w, r)$ is the unit demand for factor f in the production of good g
- Using the Envelope theorem, we can easily check that

$$a_{\mathrm{lg}}(w, r) = \frac{dc_g(w, r)}{dw} \text{ and } a_{kg}(w, r) = \frac{dc_g(w, r)}{dr}$$

- $A(w, r) \equiv [a_{fg}(w, r)]$ denotes the matrix of total factor requirements

Two-by-Two Heckscher-Ohlin Model

Equilibrium conditions (I): small open economy

- Like in RV model, we first look at the case of a "small open economy"
  - i.e. no need to look at goods market clearing
- **Profit-maximization:**

$$p_g \leqslant wa_{\mathrm{lg}}(w,r) + ra_{kg}(w,r) \text{ for all } g = 1,2 \tag{4}$$

$$p_g = wa_{\mathrm{lg}}(w,r) + ra_{kg}(w,r) \text{ if } g \text{ is produced in equilibrium} \tag{5}$$

- **Factor market clearing:**

$$l = y_1 a_{l1}(w,r) + y_2 a_{l2}(w,r) \tag{6}$$

$$k = y_1 a_{k1}(w,r) + y_2 a_{k2}(w,r) \tag{7}$$

Two-by-Two Heckscher-Ohlin Model

Factor Price Equalization (FPE)

- **Question:**

  Can trade in goods be a (perfect) substitute for trade in factors?

- First classical result from the HO literature, and the answer is affirmative
- To establish this result formally, we'll need the following definition:
- **Definition**. Factor Intensity Reversal (FIR) does not occur if:
  - (i) $a_{l1}(w,r)/a_{k1}(w,r) > a_{l2}(w,r)/a_{k2}(w,r)$ for all (w,r), or
  - (ii) $a_{l1}(w,r)/a_{k1}(w,r) < a_{l2}(w,r)/a_{k2}(w,r)$ for all (w,r)

Two-by-Two Heckscher-Ohlin Model

Factor Price Insensitivity (FPI)

- **FPI Lemma**: If both goods are produced in equilibrium and FIR does not occur, then factor prices $\omega \equiv (w, r)$ are uniquely determined by good prices $p \equiv (p_1, p_2)$
- **Proof**: If both goods are produced in equilibrium, then $p = A'(\omega)\omega$. By Gale and Nikaido (1965), this equation admits a unique solution if $a_{fg}(w) > 0$ for all f, g and det $[A(w)] \neq 0$ for all ω, which is guaranteed by no FIR.
- **Comments:**
  - Good prices rather than factor endowments determine factor prices
  - In a closed economy, good prices and factor endowments are related, but not for a small open economy
  - All economic intuition can be gained by simply looking at Leontieff case
  - Proof already suggests that "dimensionality" will be an issue for FIR

Two-by-Two Heckscher-Ohlin Model

Factor Price Equalization (FPE) Theorem

- The previous lemma directly implies the *FPE Theorem* (Samuelson 1949)
- **FPE Theorem** If two countries produce both goods under free trade with the same technology and FIR does not occur, then they must have the same factor prices
- **Comments:**
  - Important result: trade act as a perfect substitute for factor mobility
  - Countries with different factor endowments can sustain same factor prices through different allocation of factors across sectors
  - Assumptions for FPE are stronger than for FPI: we need free trade and same technology in the two countries.

Two-by-Two Heckscher-Ohlin Model

Stolper-Samuelson (1941) Theorem

- **Stolper-Samuelson Theorem** *An increase in the relative price of a good will increase the real return to the factor used intensively in that good, and reduce the real return to the other factor*
- **Proof**:
  - W.l.o.g, suppse that (i) $a_{l1}(w,r)/a_{k1}(w,r) > a_{l2}(w,r)/a_{k2}(w,r)$ and (ii) $\widehat{p}_2 > \widehat{p}_1$.
  - Differentiating the zero profit condition (5), we get

$$\widehat{p}_g = \theta_{\mathrm{lg}}\widehat{w} + (1-\theta_{lg})\widehat{r}, \tag{8}$$

where $\widehat{x} \equiv d\ln x = dx/x$ and $\theta_{\mathrm{lg}} \equiv wa_{\mathrm{lg}}(\omega)/c_g(\omega)$. Equation (8) + (ii) imply

$$\widehat{w} > \widehat{p}_2 > \widehat{p}_1 > \widehat{r} \text{ or } \widehat{r} > \widehat{p}_2 > \widehat{p}_1 > \widehat{w}$$

By (i), $\theta_{l1} > \theta_{l2}$. So (ii)+(8) further implies $\widehat{r} > \widehat{w}$. Combing the previous inequalities, we get

$$\widehat{r} > \widehat{p}_2 > \widehat{p}_1 > \widehat{w}$$

Two-by-Two Heckscher-Ohlin Model

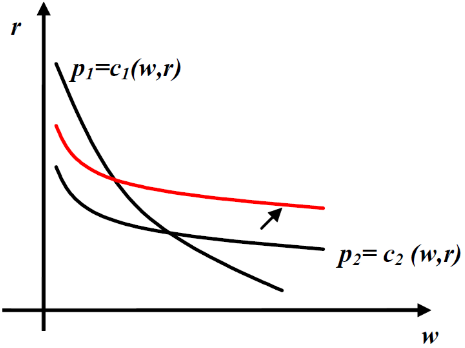
Stolper-Samuelson (1941) Theorem

Comments:

- Previous "hat" algebra is often referred to "Jones (1965) algebra"
- The chain of inequalities $\hat{r} > \hat{p}_2 > \hat{p}_1 > \hat{w}$ is referred as a "magnification effect"
- SS predict both winners and losers from change in relative prices
- Like FPI and FPE, SS entirely comes from zero-profit condition
- Like FPI and FPE, sharpness of the result hinges on dimensionality
- In the empirical literature, people often talk about "Stolper-Samuelson effects" whenever looking at changes in relative factor prices (though changes in relative good prices are rarely observed)

Two-by-Two Heckscher-Ohlin Model

Stolper-Samuelson (1941) Theorem: graphical analysis



- Like for FPI and FPE, all economic intuition could be gained by looking at the simpler Leontieff case:
  - In the general case, iso-cost curves are not straight lines, but under no FIR, same logic applies

Two-by-Two Heckscher-Ohlin Model

Robczynski (1965) Theorem

- Previous results have focused on the implication of zero profit condition, Equation (5), for *factor prices*
- We now turn our attention to the implication of factor market clearing, Equations (6) and (7), for *factor allocation*
- **Rybczynski Theorem** *An increase in factor endowment will increase* the output of the industry using it intensively, and decrease the output of the other industry

Two-by-Two Heckscher-Ohlin Model

Robczynski (1965) Theorem

Proof:

- W.l.o.g, suppse that (i) $a_{l1}(w,r)/a_{k1}(w,r) > a_{l2}(w,r)/a_{k2}(w,r)$ and (ii) $\widehat{k} > \widehat{l}$.
- Differentiating factor market clearing conditions (6) and (7) , we get

$$\begin{aligned} \widehat{l} &= \lambda_{l1}\widehat{y_1} + (1-\lambda_{l1})\widehat{y_2} \\ \widehat{k} &= \lambda_{k1}\widehat{y_1} + (1-\lambda_{k1})\widehat{y_2} \end{aligned} \tag{9}$$

where $\lambda_{l1} \equiv a_{l1}(\omega)y_1/l$ and $\lambda_{k1} \equiv a_{k1}(\omega)y_1/k$ Equation (9) + (ii) imply

$$\widehat{y_1} > \widehat{k} > \widehat{l} > \widehat{y_2} \text{ or } \widehat{y_2} > \widehat{k} > \widehat{l} > \widehat{y_1}$$

By (i), $\lambda_{l1} > \lambda_{k1}$. So (ii)+(9) further implies $\widehat{y_2} > \widehat{y_1}$. Combing the previous inequalities, we get

$$\widehat{y_2} > \widehat{k} > \widehat{l} > \widehat{y_1}$$

Two-by-Two Heckscher-Ohlin Model

Robczynski (1965) Theorem

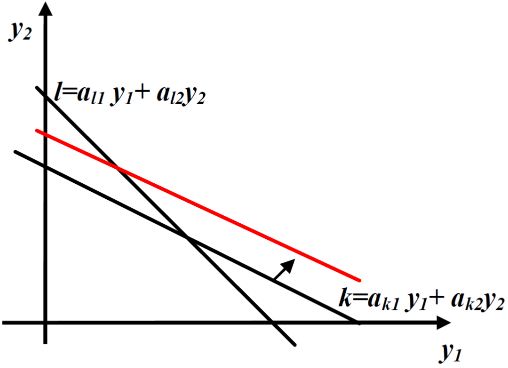
Comments:

- Like for FPI and FPE Theorems:
  - (p_1, p_2) is exogenously given $\implies$ factor prices and factor requirements are not affected by changes in factor endowments
  - Empirically, Rybczynski Theorem suggests that impact of immigration may be very different in closed vs. open economy
- Like for SS Theorem, we have a "magnification effect"
- Like for FPI, FPE, and SS Theorems, sharpness of the result hinges on "dimensionality"

Two-by-Two Heckscher-Ohlin Model

Robczynski (1965) Theorem: graphical analysis

- Since good prices are fixed, it is as if we were in Leontieff case



Heckscher-Ohlin Theorem

- We now turn to a world economy with two countries, North and South
- We maintain the two-by-two HO assumptions:
    - Two goods, $g = 1, 2$, and two factors, k, l
    - Identical technology around the world, $y_g = f^g(l_g, k_g)$
    - Identical homothetic preferences around the world, $d_g^c = \alpha_g(p)I^c$
    - Different factor endowments, summarized by the endowment vectors (v^s, v^n)
- **Question:** What is the pattern of trade in this environment?

Heckscher-Ohlin Theorem

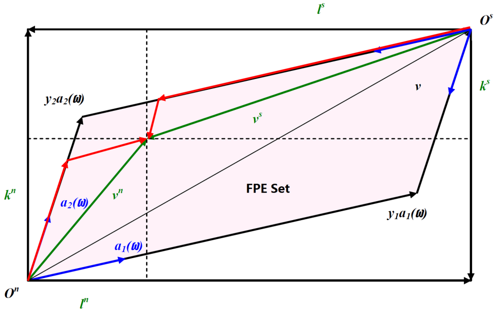
Integrated equilibrium

- If we divide the world into two countries, H and F, with $v^H + v^F = v^w$. What devisions of world endowments v^w lead to a trade equilibrium in which prices, factor rewards, aggregate output and employment levels are the same as in a equilibrium of a single country?
- This type of trading equilibrium is called an "*integrated equilibrium*".

Heckscher-Ohlin Theorem

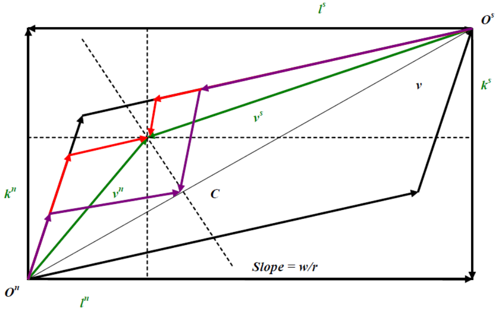
The FPE set

- The set of vectors (v^H, v^F) that leads to an integrated equilibrium can be described by the factor equalization set, or FPE set.



Heckscher-Ohlin Theorem

- Suppose that (v^s, v^n) is in the FPE set (i.e. factor price equalization holds)
- **HO Theorem** *In the free trade equilibrium, each country will export the good that uses its abundant factor intensively*



- Outside the FPE set, additional technological and demand considerations matter

High-Dimensional Predictions

Heckscher-Ohlin-Vanek

- Many countries, $c = 1, ..., i$
- Many industries (goods), $g = 1, ..., n$
- Many factors, $f = 1, ..., m$
- Identical technologies and preferences across countries, factor price equalization
- Technology A $m \times n$ technology matrix which contain all unitary inputs

  Ex: 2×2 model: $A = \begin{pmatrix} a_{l1} & a_{l2} \\ a_{k1} & a_{k2} \end{pmatrix}$

- y^c vector $(n \times 1)$ of output of each good in country c
- d^c vector $(n \times 1)$ of demand of each good in country c
- Vector of net export flows: $t^c = y^c - d^c$
- **Net factor content of trade** $(m \times 1$ vector)**:** $f^c = At^c$

High-Dimensional Predictions

Heckscher-Ohlin-Vanek

- Homothetic tastes + free trade → consumption vectors in all countries should be proportional to each others: $d^c = s^c d^w$
  where s^c is country c's share of world consumption and d^w is the vector of world consumption - equal to world production
- Hence $t^c = y^c - s^c d^w = y^c - s^c y^w \rightarrow At^c = Ay^c - s^c Ay^w$

$\Longrightarrow$ **HOV theorem.** $\boxed{f^c = v^c - s^c v^w}$

- If a country c's endowment of factor k exceeds c's share of world output, we say that country c is abundant in this factor and the factor content of trade in this factor should be positive (c must be a net exporter of the services of this factor)
- Country c is **abundant in factor** f if $\frac{v_f^c}{v_f^w} > s^c$

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