Technical Note

Yuan Zi

Dixit Stiglitz Model

Compare Competitive Equilibrium with Social Optimum

Monopolistic competition implies that the competitive equilibrium is not necessarily Pareto optimal.

- The model exhibits a version of the aggregate demand externalities:

  a. There is a markup over the marginal cost of production

  b. The number of output produced may not be optimal

- The first inefficiency is familiar from models of static monopoly

- while the second emerges from the fact that in this economy the set of commodities is endogenously determined

This relates to the issue of endogenously incomplete markets (there is no way to purchase an input that is not supplied in equilibrium).

Consider the Dixit-Stiglitz (1977)'s original discussion:

Model Setup

- The utility maximization problem of the consumer is given by:

$$\max_{q_0, q_j} U = U(q_0, Q)$$

  where

$$Q = \left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$$s.t. \quad \sum_{j=1}^{N} q_j p_j + p_0 q_0 = L \equiv E,$$

where the wage is normalized to 1.

- The marginal cost of production of q_0 is 1, the market for q_0 is of perfect competition.
- The marginal cost of production of q_i is c, the fixed cost is f.

Competitive Equilibrium

a. Numeraire good:

$$p_0 = 1$$

b. Differentiated good:

$$p_i = \frac{c}{1-\frac{1}{\sigma}}; \; P^{1-\sigma} = \sum_{k=1}^{N} p_k^{1-\sigma} \;\Rightarrow P = N^{\frac{1}{1-\sigma}} p \;\text{ under symmetry.}$$

Under free entry

$$\frac{1}{\sigma} p_i q_i = f \;\Rightarrow q_i = \frac{(\sigma-1)f}{c}.$$

$\rightarrow$ because of free entry condition, each variety's equilibrium quantity is fixed.

c. At the aggregate, we have (i). Price equals marginal utility; (ii) Budget constraint holds

$$\frac{U_Q}{U_0} = \frac{P}{1}; \; PQ + q_0 = L$$

Optimal Allocation

- Social Planer's problem

$$\max_{q_0, q_j} U = U(q_0, Q) \quad where \quad Q = \left[\sum_{j=1}^{N} q_j^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$$s.t. \quad \sum_{j=1}^{N}(cq_j + f) + q_0 = L,$$

Under symmetry this can be rewritten as:

$$\max_{q_0, q_j} U = U(q_0, qN^{\frac{\sigma}{\sigma-1}})$$

$$s.t. \quad N(cq + f) + q_0 = L,$$

- FOC:

$$U_0 = \lambda; \tag{1}$$

$$U_Q \cdot N^{\frac{\sigma}{\sigma-1}} = \lambda \cdot Nc \tag{2}$$

$$U_Q \cdot \frac{\sigma}{\sigma-1} qN^{\frac{\sigma}{\sigma-1}-1} = \lambda \cdot (cq + f) \tag{3}$$

$$Ncq + Nf + q_0 = L \tag{4}$$

- Because of (2) and (3):

$$\frac{\sigma}{\sigma-1} q = \frac{cq+f}{c} \quad \Rightarrow \quad q_i = \frac{(\sigma-1)f}{c} \tag{5}$$

→ Social planner will choose the same optimal quantity per variety. Because free entry condition, in competitive equilibrium "no excess profits" act liked a social planner who takes into account the total cost of producing q_i (variable + fixed costs).

- Because of (1) and (2):

$$\frac{U_Q}{U_0} = \frac{N^{\frac{1}{1-\sigma}} c}{1};$$

Compared to in the competitive equilibrium:

$$\frac{U_Q}{U_0} = \frac{N^{\frac{1}{1-\sigma}} p}{1}.$$

Clearly Q needs to be greater under the social planer case (U_Q smaller). In other words, positive markup distorted (reduced) the consumption of the differentiated aggregate.

→ Social planner will choose a greater number of varieties.

Take-away Lessons

a. Monopolistic competition *can* lead to too little entry. This is due positive markups. The "composite aggregate" has the same flavor as a monopoly.

b. Free entry can, to some extent, "help" in reaching optimality – the quantity per variety is at the social optimal.

c. The distortion, directly speaking, comes from higher-than-marginal-cost price of Q, vs marginal-cost price of q_0 charging. If no markup on varieties, or same markup for numeraire goods, socially optimal allocation can be reached. If there is no numeraire goods, the competitive equilibrium is also Pareto optimal – in this case labor market clearing ensures optimal number of N.

d. It is really the mark-up wedge + free entry conditions that joint decide if the competitive allocation is optimal or not.

Overall:

"There is an important warning here: one has to be very careful about making welfare statements in trade, macroeconomic, and growth models using the Dixit–Stiglitz framework. If one obtains a result that the market is inefficient, the analysis can be useful, in isolating another market failure. If one obtains a result that the market is in some sense constrained Pareto efficient, take it with a grain of salt."

– Stiglitz, J. E. (2017). Monopolistic competition, the Dixit–Stiglitz model, and economic analysis. Research in Economics, 71(4), 798-802.